A Tropospheric Delay Model for the user of the Wide Area Augmentation System

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INTRODUCTION

- OBJECTIVES
  - ➡ Develop and test a tropospheric propagation delay model for the WAAS user.
  - ➡ Determine the bounds of the tropospheric delay contribution to the pseudorange error budget.
- LIMITATION
  - ➡ Lack of "real-time" meteorology to quantify the state of the atmosphere.

TROPOSPHERIC DELAY

- Refractive index in neutral atmosphere greater than unity.
- Decrease in velocity increases propagation time and equivalent path length (the "delay").
- Refraction bends raypath - significant at low elevation angles.
- Hydrostatic zenith delay accurately determined with atmospheric pressure measurement.
- Ignoring horizontal gradients and assuming azimuthal symmetry allows use of mapping functions (ratios of zenith delay to angle of elevation delay).

DELAY FORMULATION

Zenith Delay, $$d_{trop}^{z} = 10^{-6} \int_{r_0}^{r_1} N \, dr$$

N - refractivity

r_0, r_1 - radial distance from centre of earth

Refractivity, $$N = k_1 \frac{P_d}{T} + k_2 \frac{e}{T} + k_3 \frac{e}{T^2}$$

k_1, k_2, k_3 - refractivity constants

P_d, T, e - atmospheric parameters

Total Delay, $$d_{trop} = d_{hyd}^{z} \cdot m_{hyd} + d_{wet}^{z} \cdot m_{wet}$$

d_{hyd}^{z}, d_{wet}^{z} – hydrostatic and wet zenith delay,

m_{hyd}, m_{wet} – hydrostatic and wet mapping function

NAVIGATION-TYPE (NAV) MODELS

	Elevation Angle	Latitude	Height	Met. Parameters
Altshuler	✓	✓	✓	N
WAAS	✓	✓	✓	N
NATO†	✓	✗	✓	N
UNB1‡	✓	✓	✓	P, T, e, β, λ

† Central Radio Propagation Laboratory Reference Atmosphere 1958
+ Chao 'dry' Mapping Function

‡ Saastamoinen Zenith Delay models + Niell Mapping Functions +

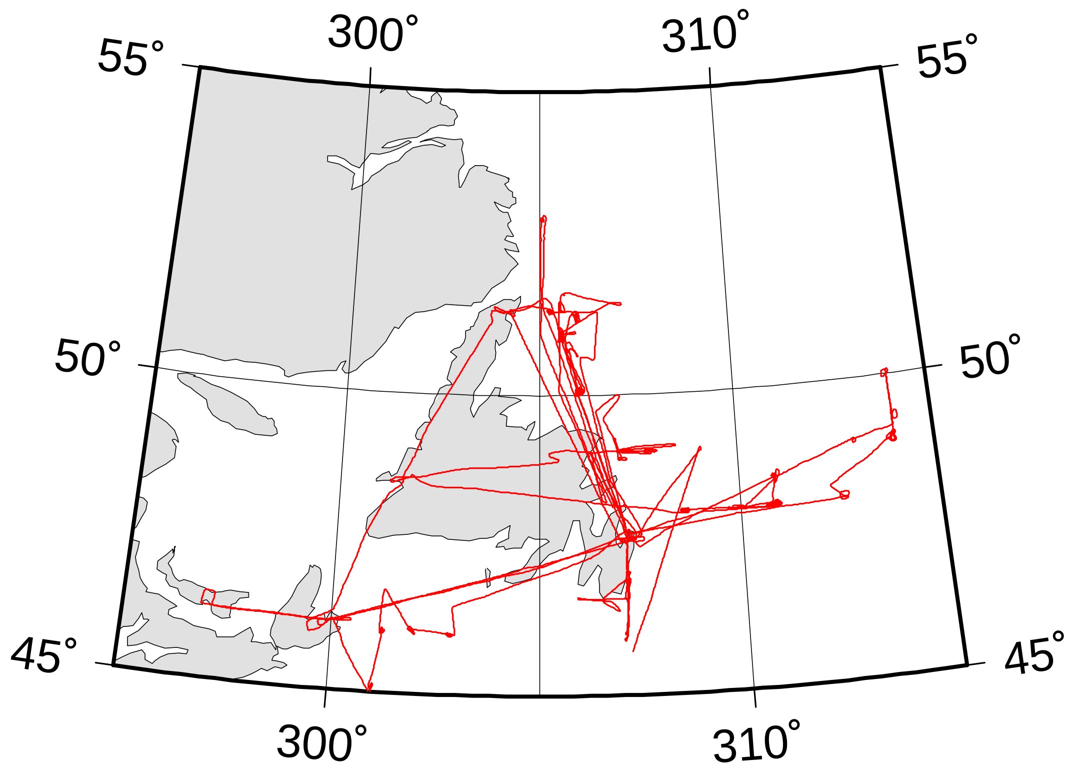
$$T = T_0 - \beta \cdot H, \qquad P = P_0 \left(\frac{T}{T_0} \right)^{\frac{g}{R\beta}}, \qquad e = e_0 \left(\frac{T}{T_0} \right)^{\frac{4g}{R\beta}},$$

$$f\,(\,T_0 = 288.15\text{K},\ P_0 = 1013.25\text{mbar},\ e_0 = 11.7\text{mbar}\,) \approx 324.8 \text{ N units}$$

DATA DESCRIPTION AND PROCESSING

- Frizzle '95 Experiment, Newfoundland, Canada, March 1995.
- 12, 3-5 hour flights recording dual frequency GPS and meteorological data at aircraft and ground station.
- P(Y) code on L1 processed for optimum positions.
- IGS precise orbits used.
- Ionosphere largely removed with single differences.
- Benchmark solutions computed using UNB1 and the simultaneously recorded meteorological data.

FRIZZLE '95 FLIGHT PATHS



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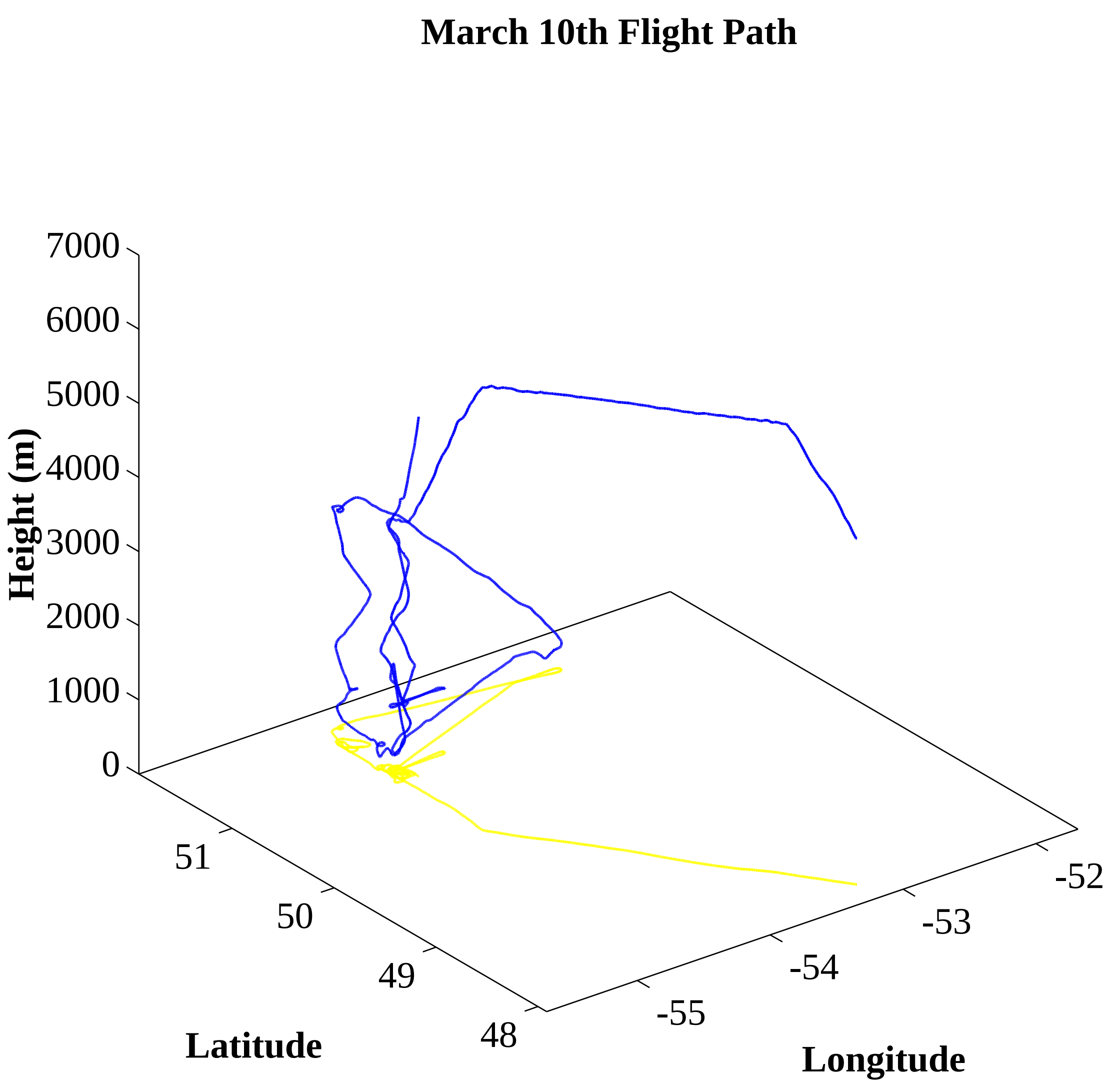
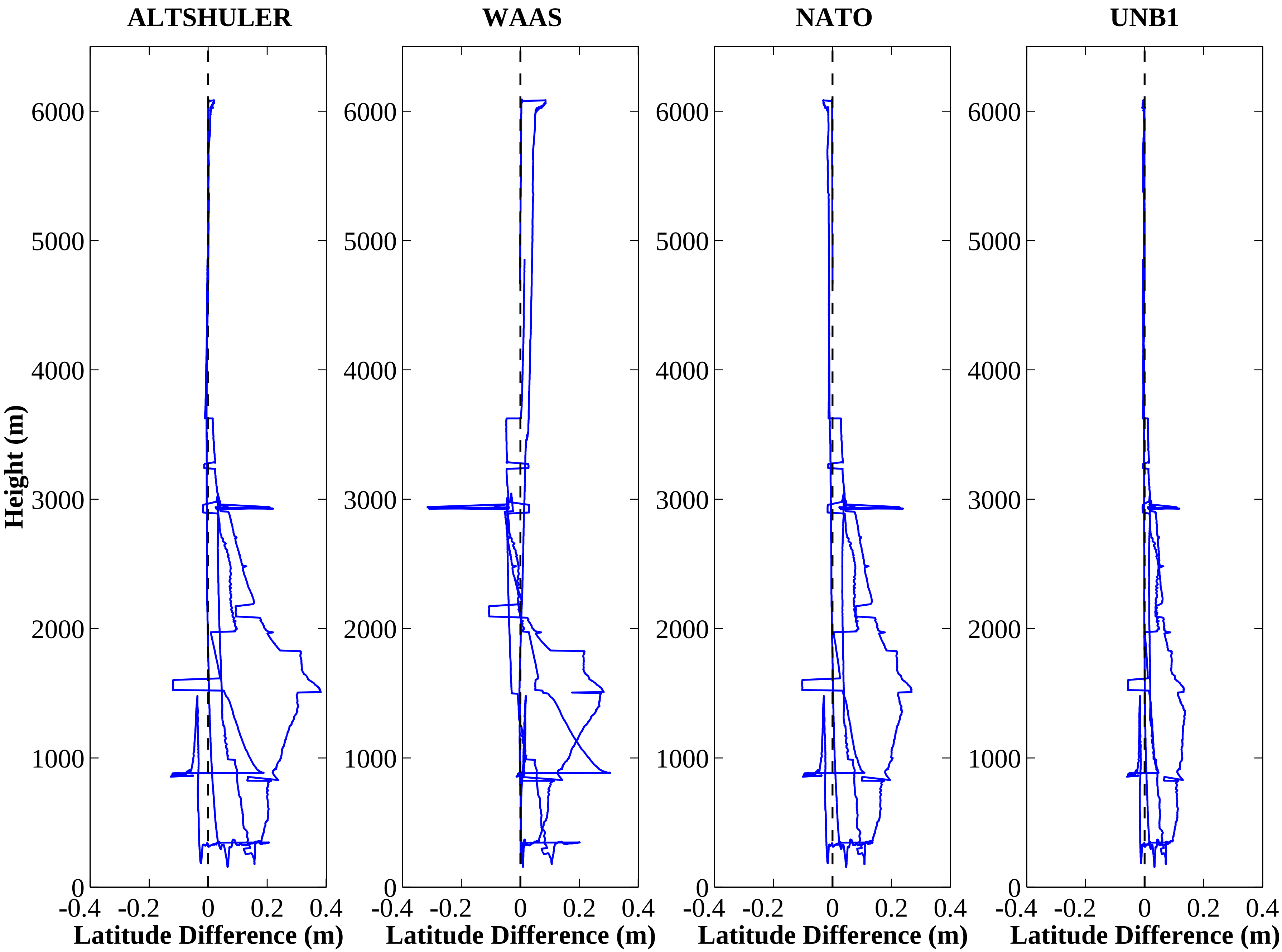
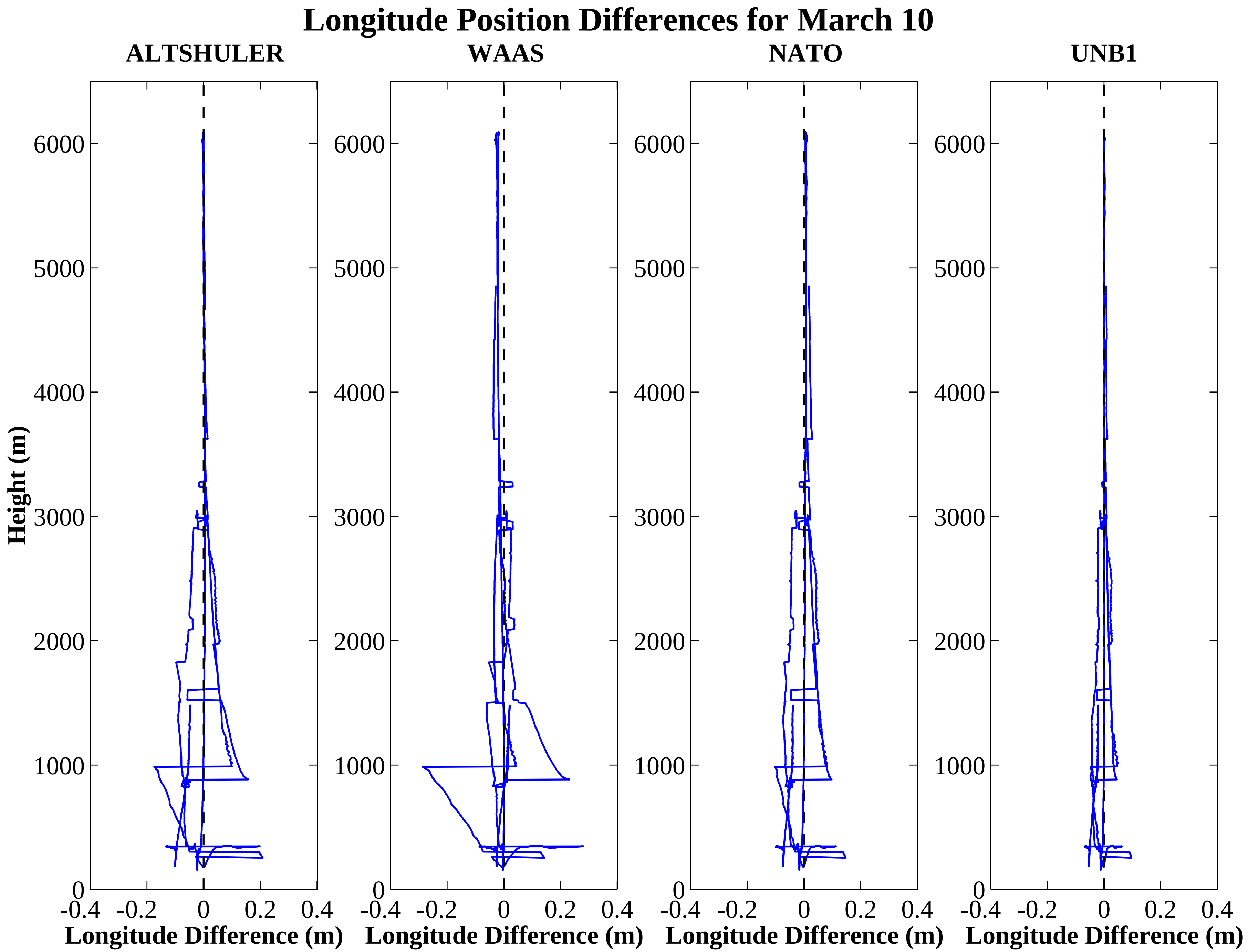
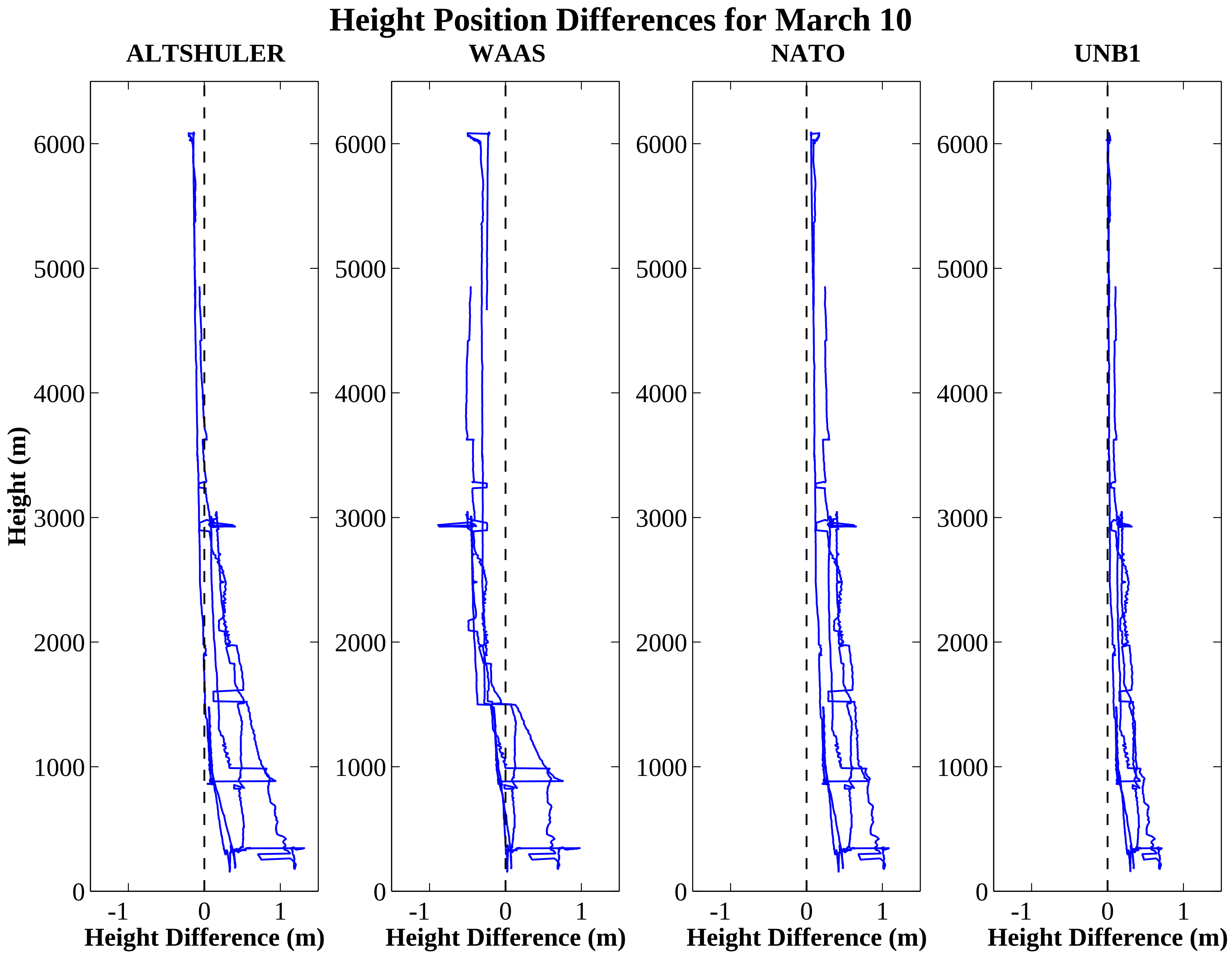

March 10th Flight Path
Height (m)
7000
6000
5000
4000
3000
2000
1000
0
51
50
49
48
Latitude
-55
-54
-53
-52
Longitude

Latitude Position Differences for March 10

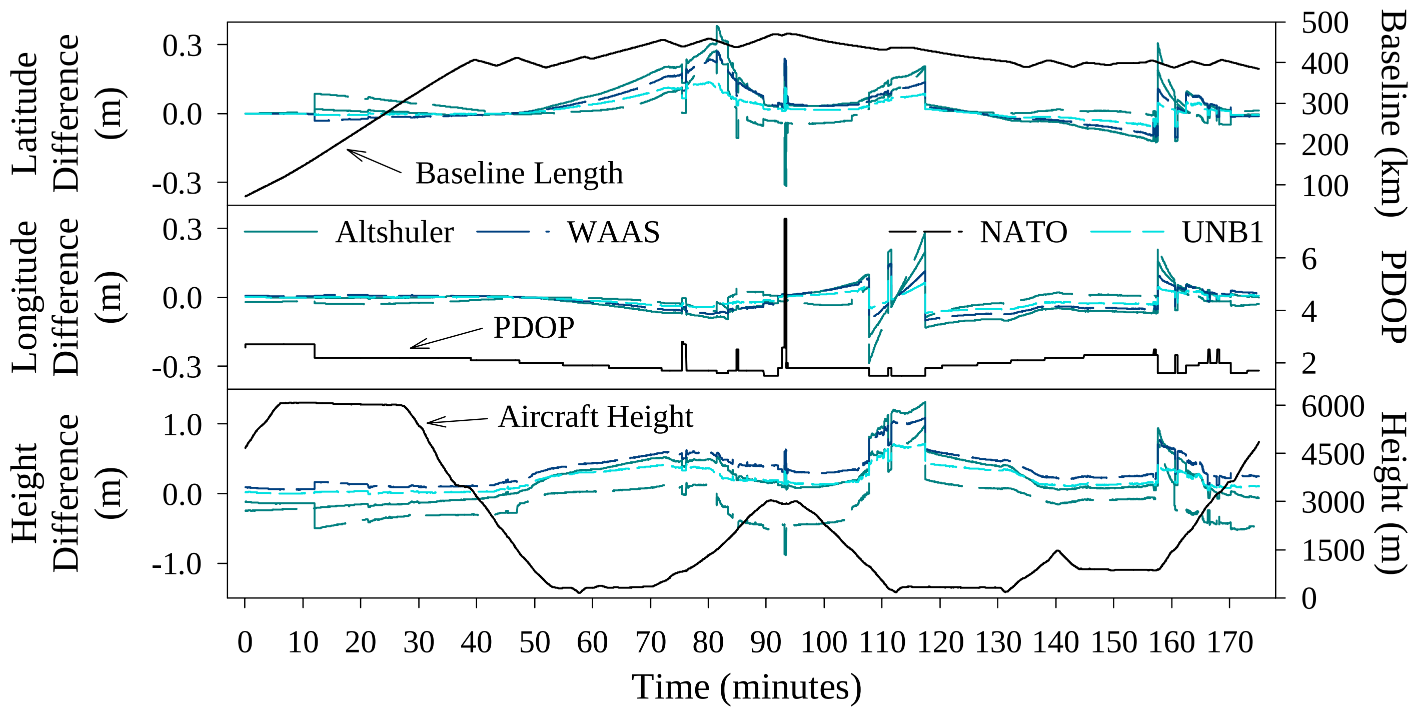




Longitude Position Differences for March 10
ALTSHULER
WAAS
NATO
UNB1
Height (m)
6000
5000
4000
3000
2000
1000
0
-0.4
-0.2
0
0.2
0.4
Longitude Difference (m)

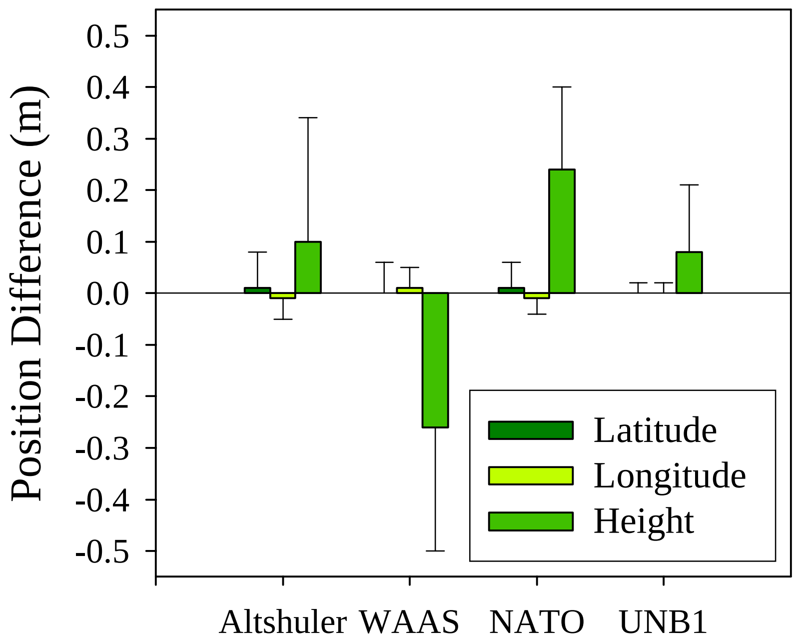

Height Position Differences for March 10
ALTSHULER
WAAS
NATO
UNB1
Height (m)
6000
5000
4000
3000
2000
1000
0
-1
0
1
Height Difference (m)

SOLUTION DIFFERENCES

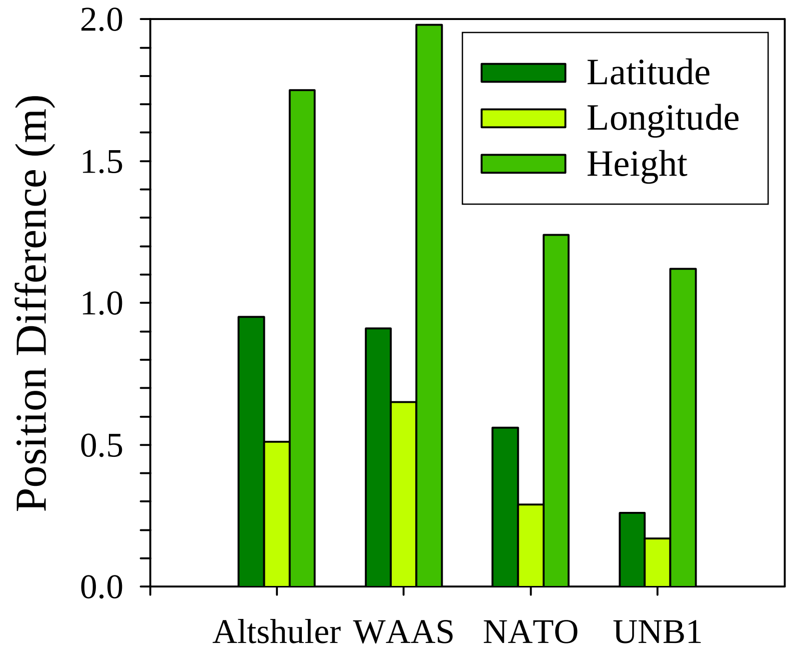


NAV GPS RESULTS

Position Difference Mean and Standard Deviation



Position Difference Ranges



NAV GPS SUMMARY

- Worst case height differences:
  - ➡ Altshuler - 1.38m.
  - ➡ WAAS - 1.07m.
  - ➡ NATO - 1.09m.
  - ➡ UNB1 - 0.71m.
- Step in WAAS solutions.
- NATO and UNB1 perform similarly.
- Altshuler and WAAS:
  - ➡ Poor modelling of height change of troposphere delay.
  - ➡ Poor elevation angle modelling.

TROPOSPHERIC HEIGHT DEPENDENCY

Hyd. Zenith Delay

$$d^z_{hyd} = f(P)$$
$$= \tau^z_{hyd} \cdot P$$

At user's altitude (H)

$$T = T_0 - \beta \cdot H$$

Wet Zenith Delay

$$d^z_{wet} = f(e, T, \beta, \lambda')$$
$$= \tau^z_{wet} \cdot \frac{e}{T}$$

$$P = P_0 \left(\frac{T}{T_0} \right)^{\frac{g}{R\beta}}$$
$$\equiv P_0 \cdot \kappa_{hyd}$$

$$\frac{e}{T} = \frac{e_0}{T_0} \left(\frac{T}{T_0} \right)^{\frac{\lambda' g}{R\beta} - 1}$$
$$\equiv \frac{e_0}{T_0} \cdot \kappa_{wet}$$

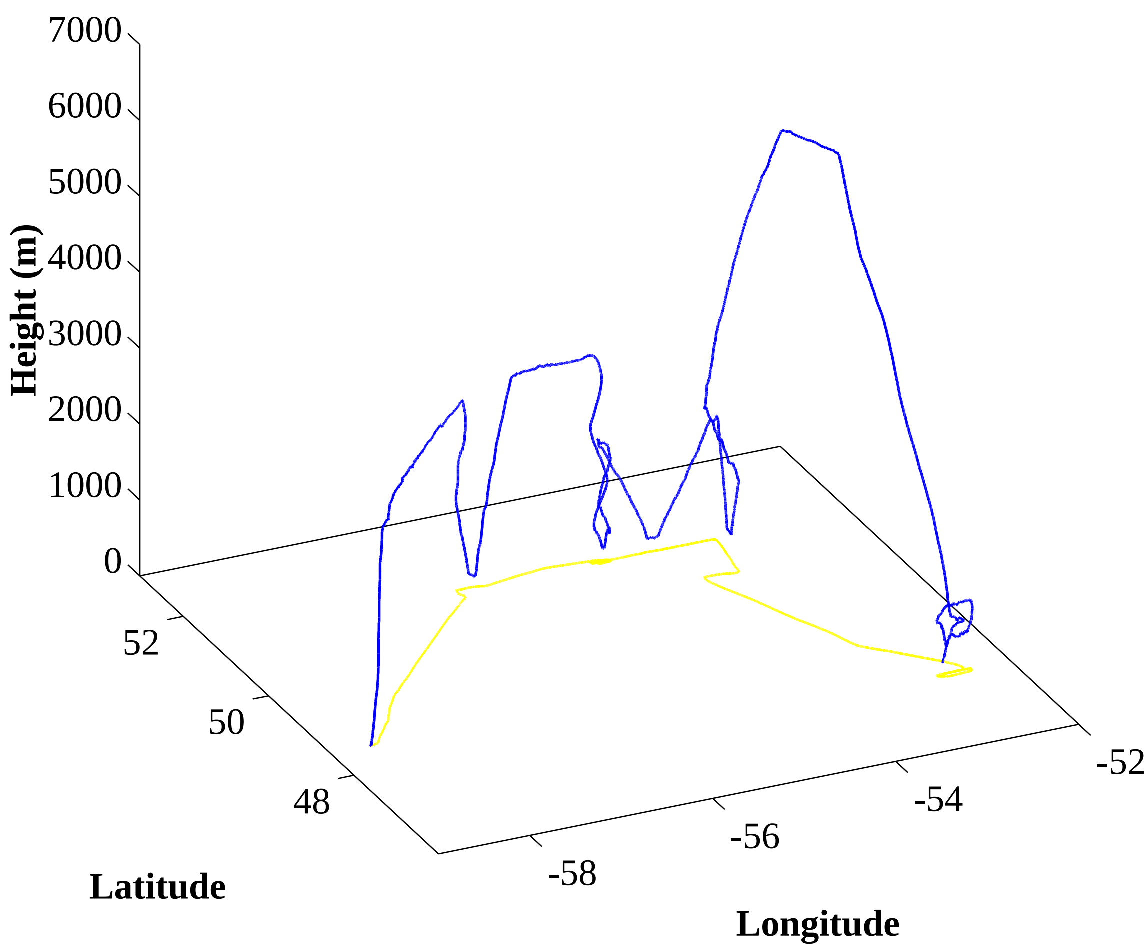
$$d^z_{hyd} = \tau^z_{hyd} \cdot \kappa_{hyd} \cdot P_0$$

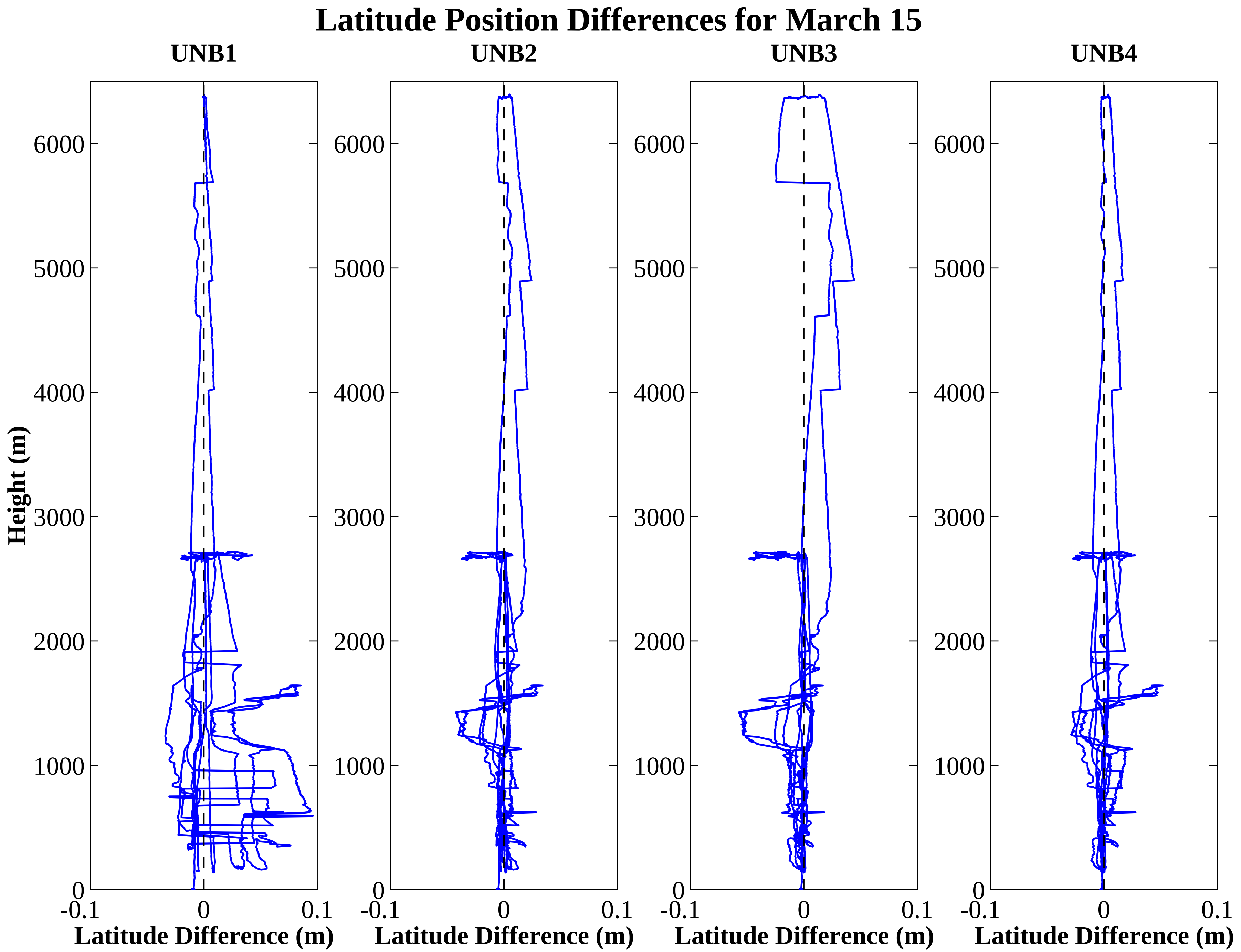
$$d^z_{wet} = \tau^z_{wet} \cdot \kappa_{wet} \cdot \frac{e_0}{T_0}$$

UNB MODEL SUMMARY

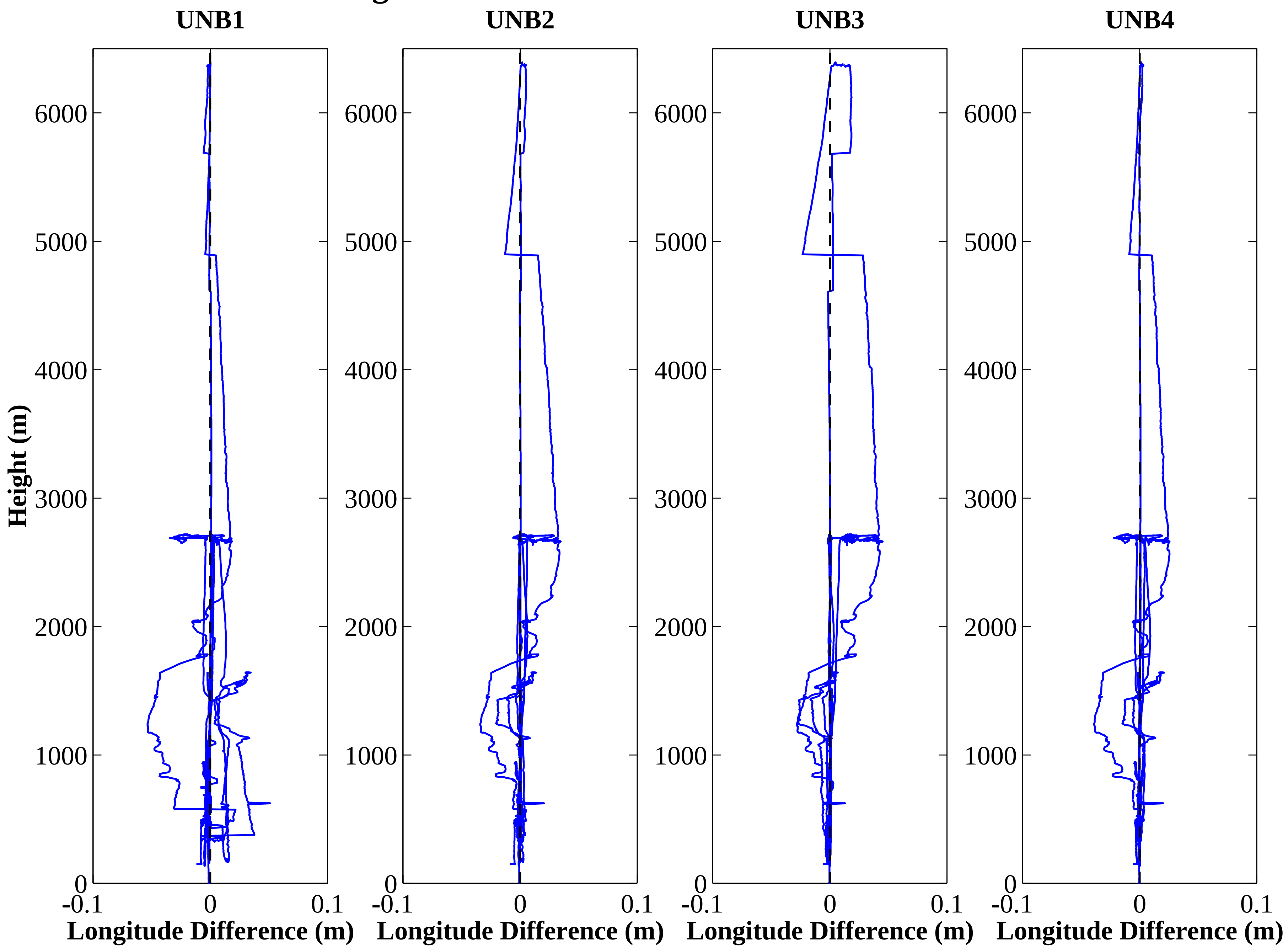
- Require values for:
  - ➡ Surface pressure (P_0),
  - ➡ Surface temperature (T_0),
  - ➡ Surface water vapour pressure (e_0),
  - ➡ Temperature lapse rate (β), and
  - ➡ Water vapour "lapse rate" (λ).
- UNB1 - Global constant values (STP).
- UNB2 - Latitudinal averages (various sources).
- UNB3 - U.S. Standard Atmosphere Supplements, 1966.
  - ➡ (temporal variation based on Niell mapping function concepts).
- UNB4 - Modification to UNB3 temperature profile.

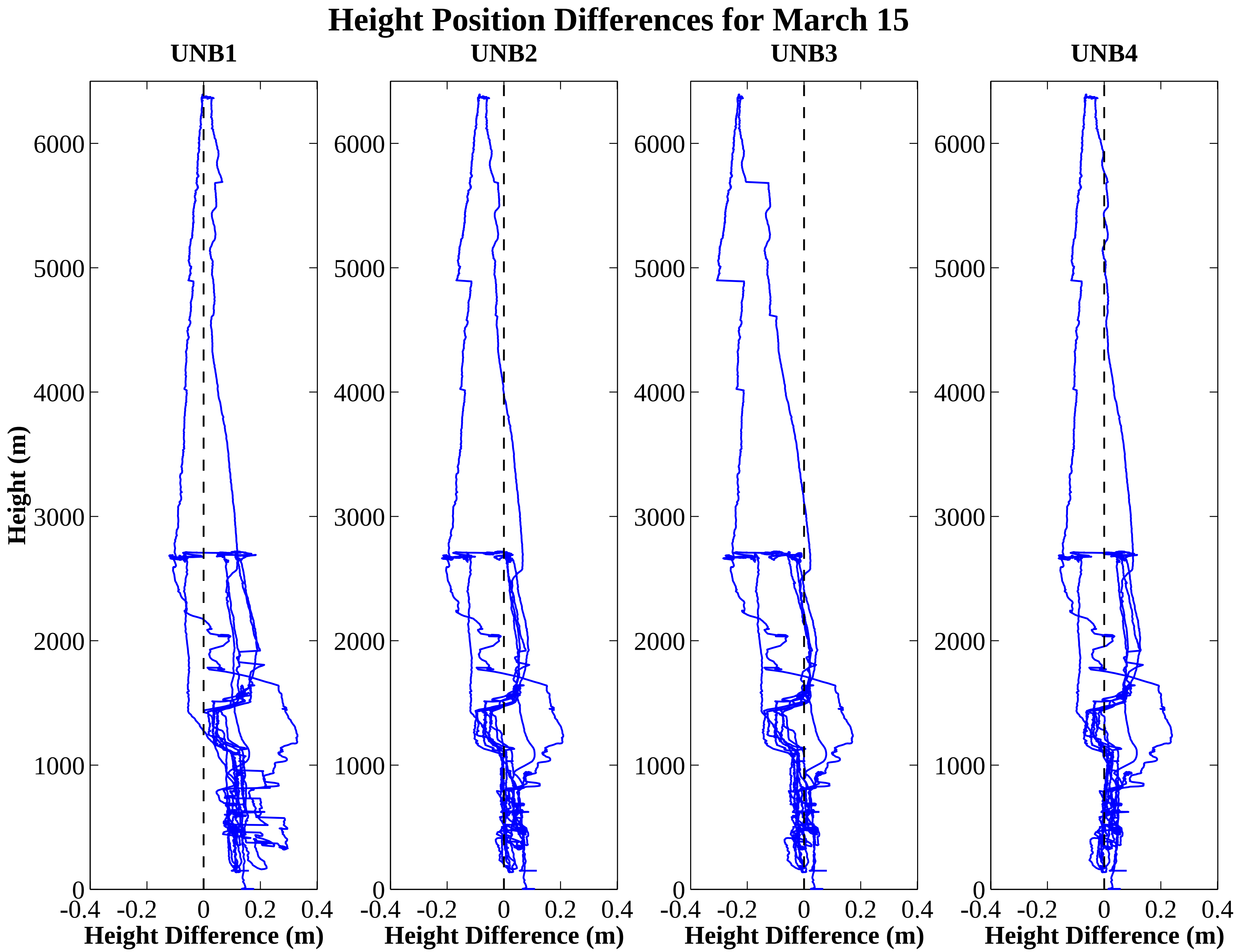
March 15th Flight Path




Latitude Position Differences for March 15
UNB1
UNB2
UNB3
UNB4
Height (m)
0
1000
2000
3000
4000
5000
6000
-0.1
0
0.1
Latitude Difference (m)

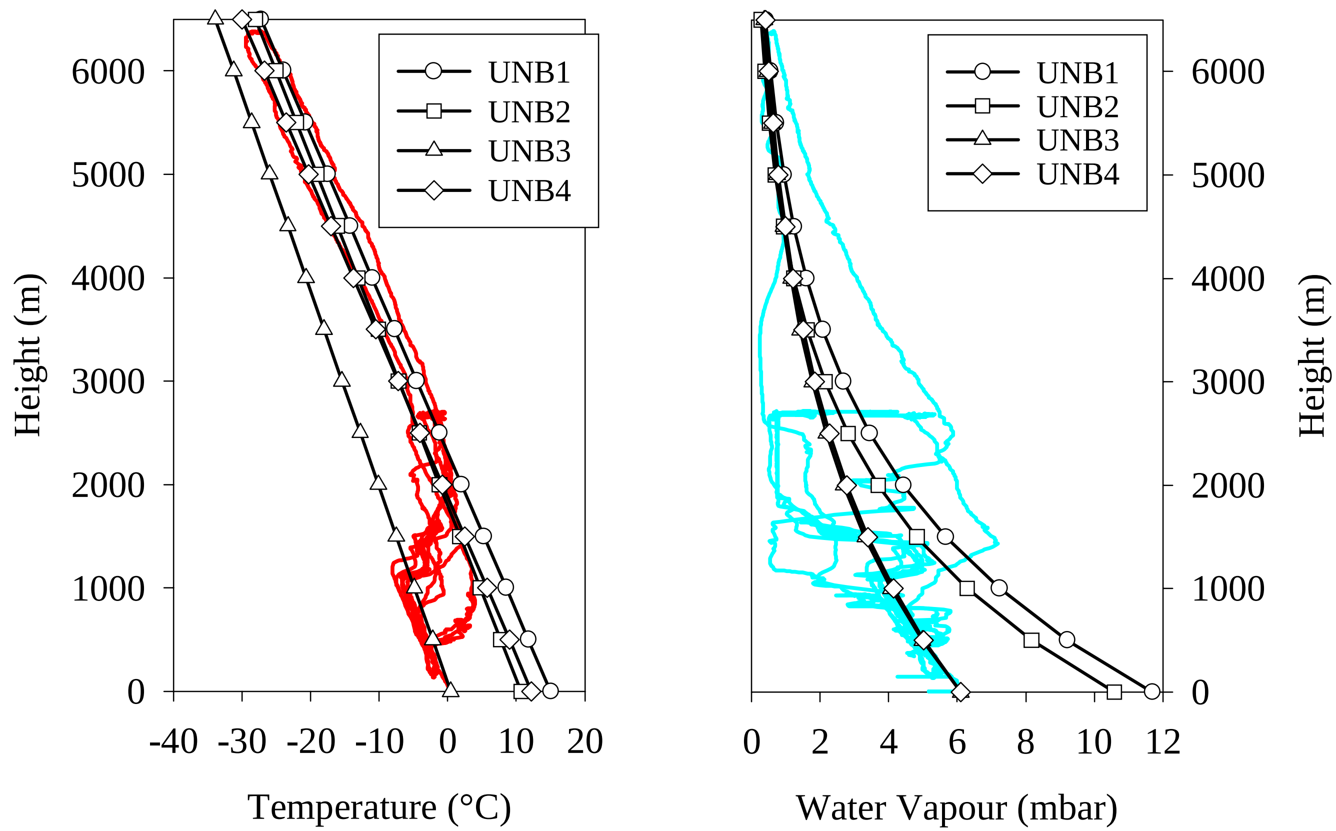
Longitude Position Differences for March 15




Height Position Differences for March 15
UNB1
UNB2
UNB3
UNB4
Height (m)
6000
5000
4000
3000
2000
1000
0
-0.4
-0.2
0
0.2
0.4
Height Difference (m)

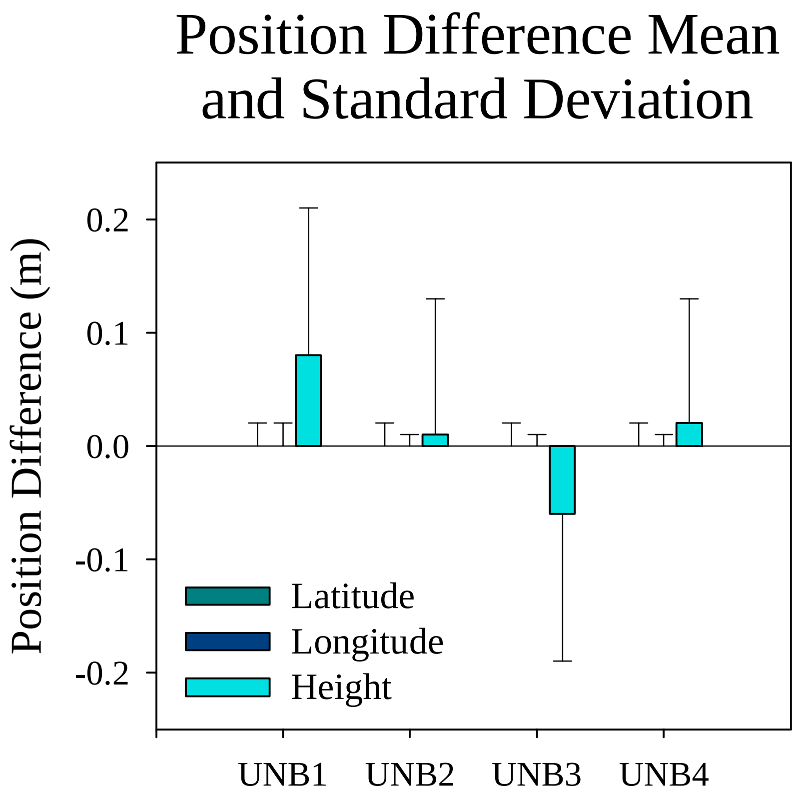
METEOROLOGICAL PROFILES

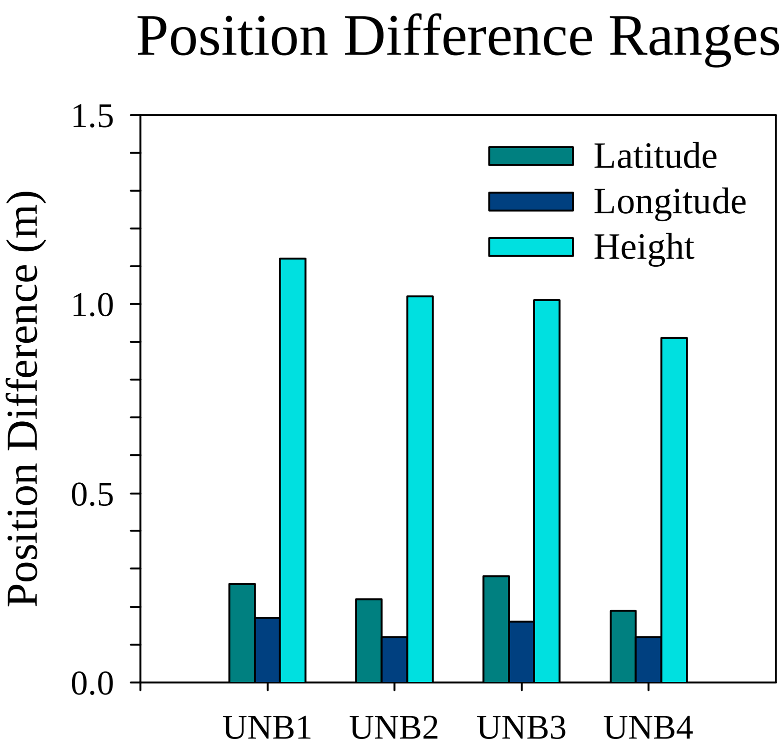
March 15



UNB GPS RESULTS (1)

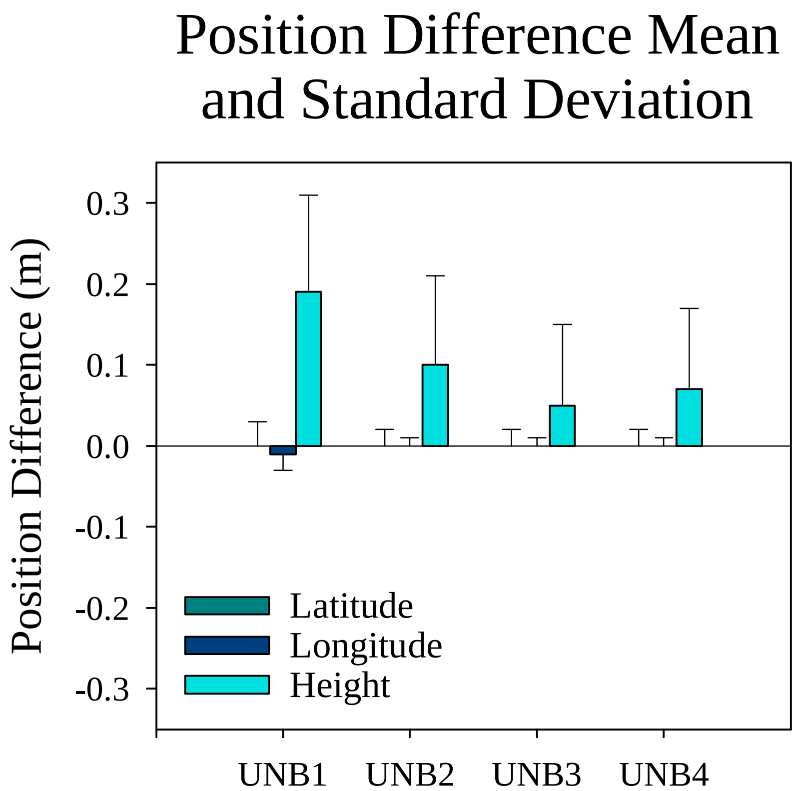
All data

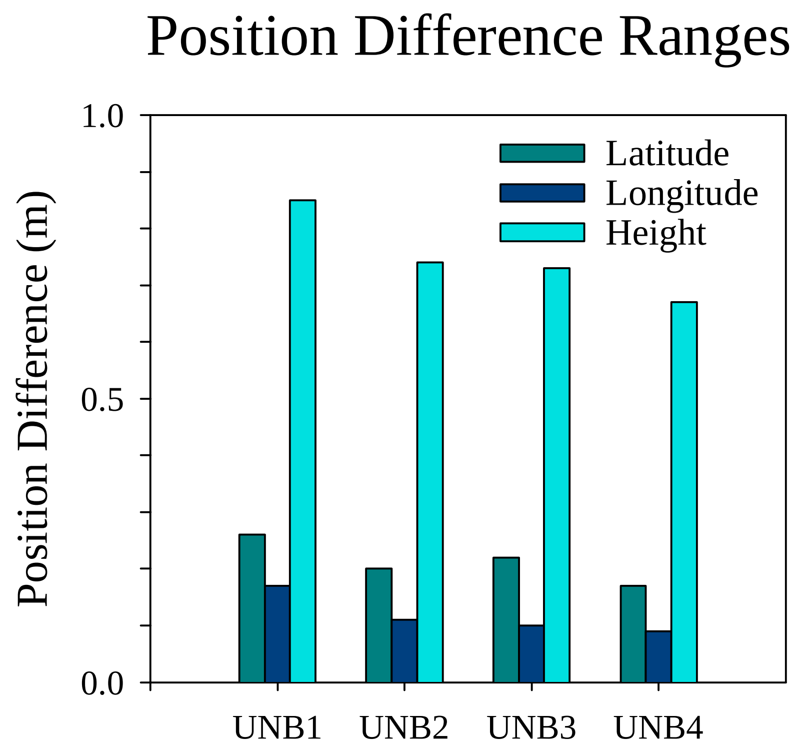




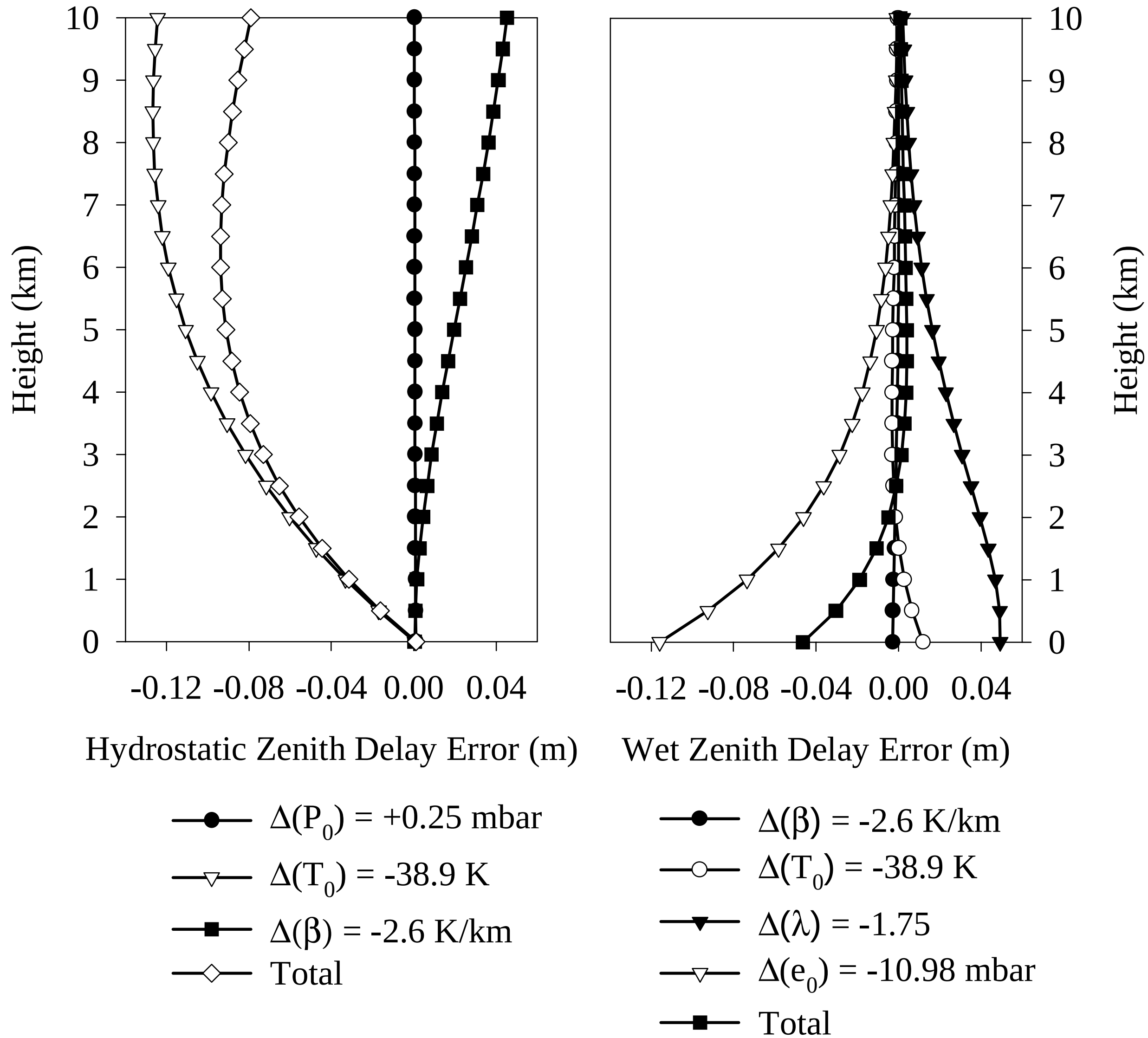
UNB GPS RESULTS (2)

Data upto 1 km in altitude (above msl)



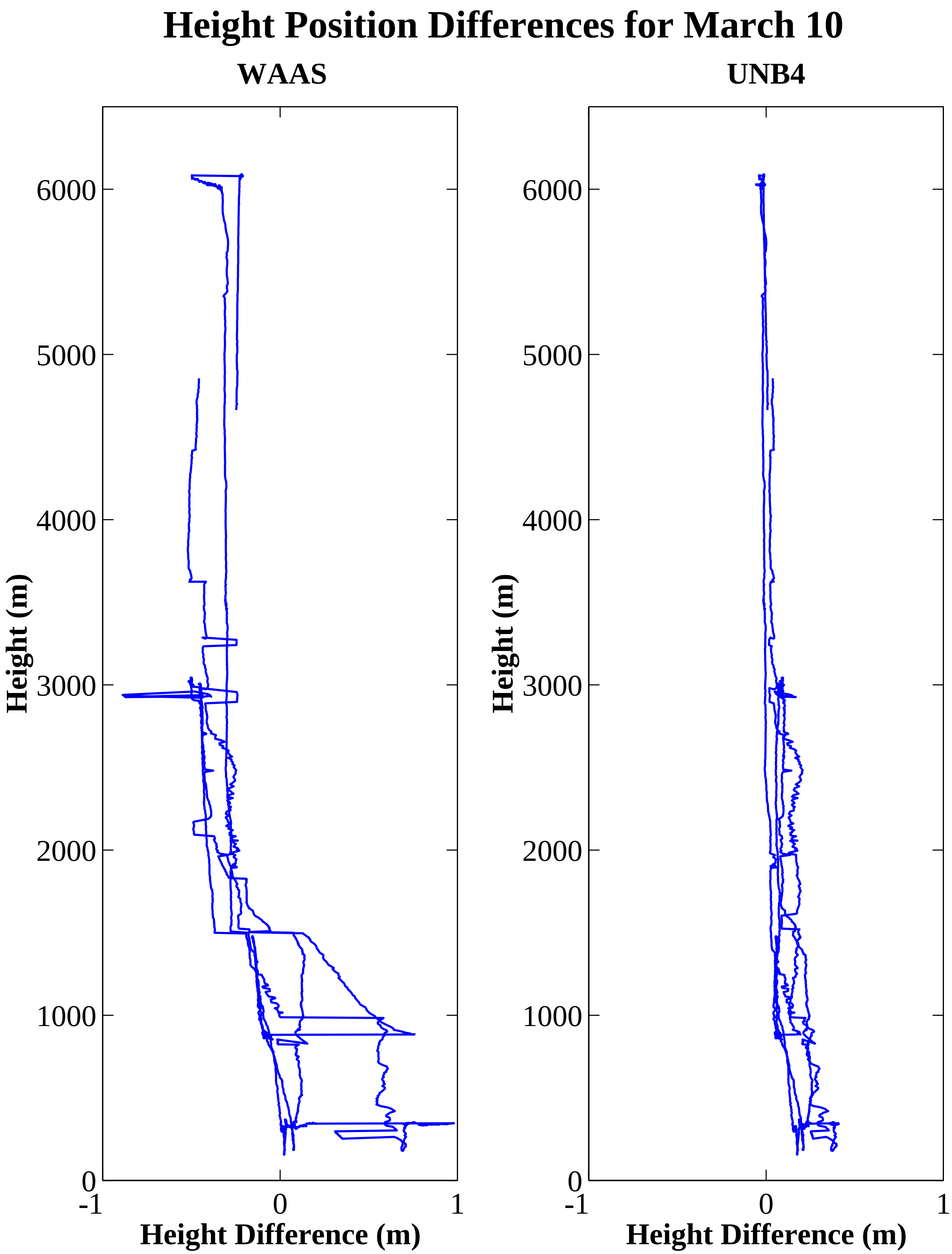


ERROR MODELLING



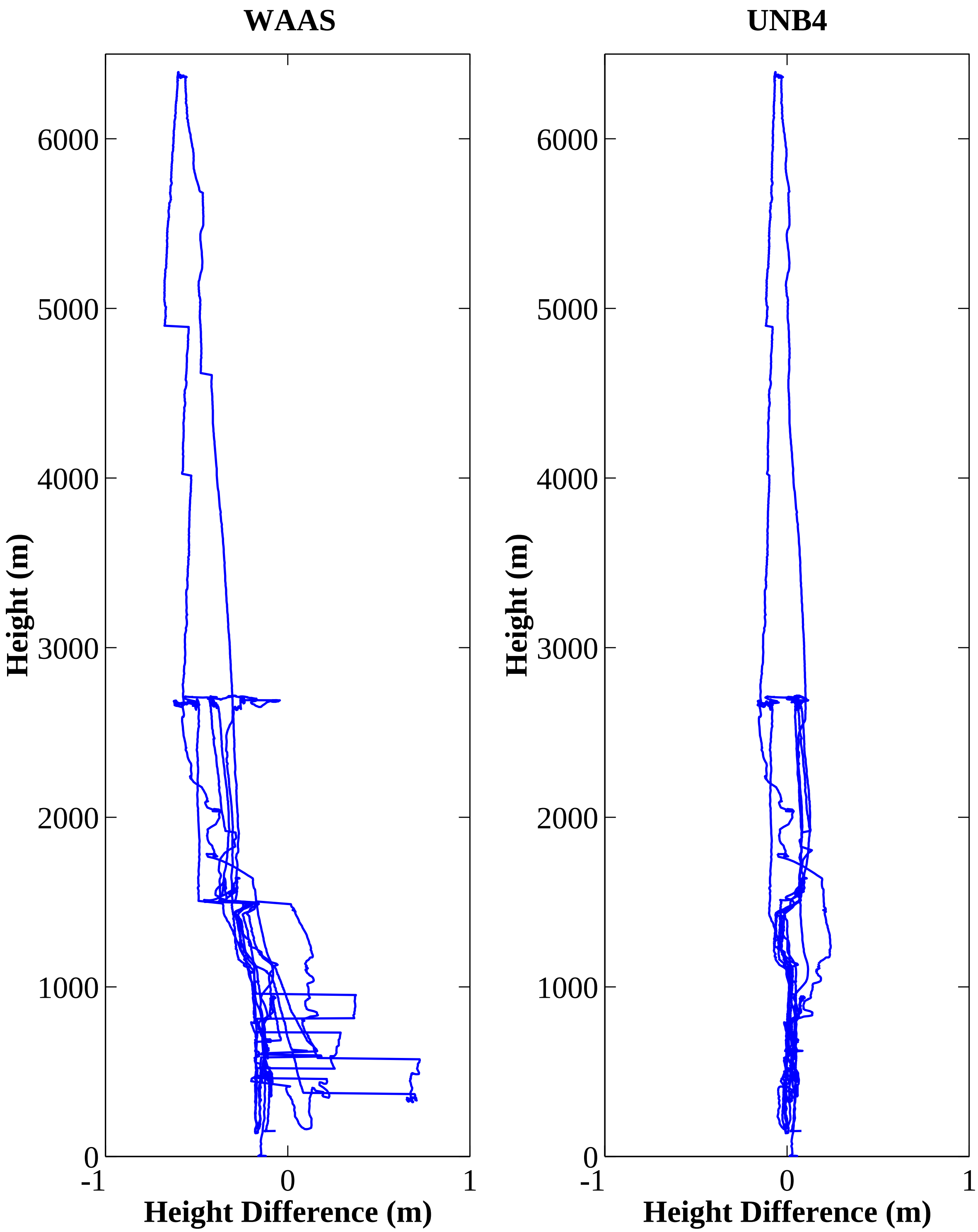
UNB GPS SUMMARY

- Error model shows potential impact of:
  - ➡ incorrect water vapour pressure at surface,
  - ➡ incorrect scaling of total surface pressure.
- Improvement in position solutions follows improved representation of water vapour profile.
- Most near-surface biases reduced by UNB3 and UNB4.
- Altitude bias introduced by UNB3.
- Most consistent results throughout flight paths provided by UNB4.

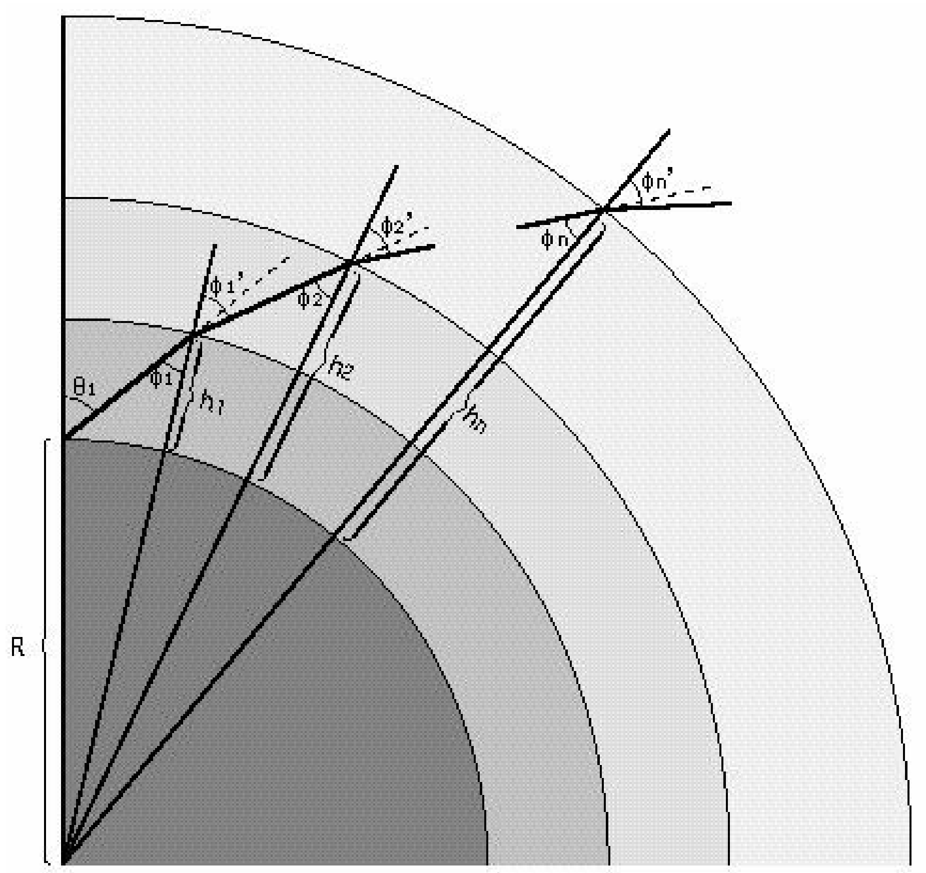


Height Position Differences for March 10
WAAS
UNB4
Height (m)
Height Difference (m)
0
1000
2000
3000
4000
5000
6000
-1
0
1

Height Position Differences for March 15



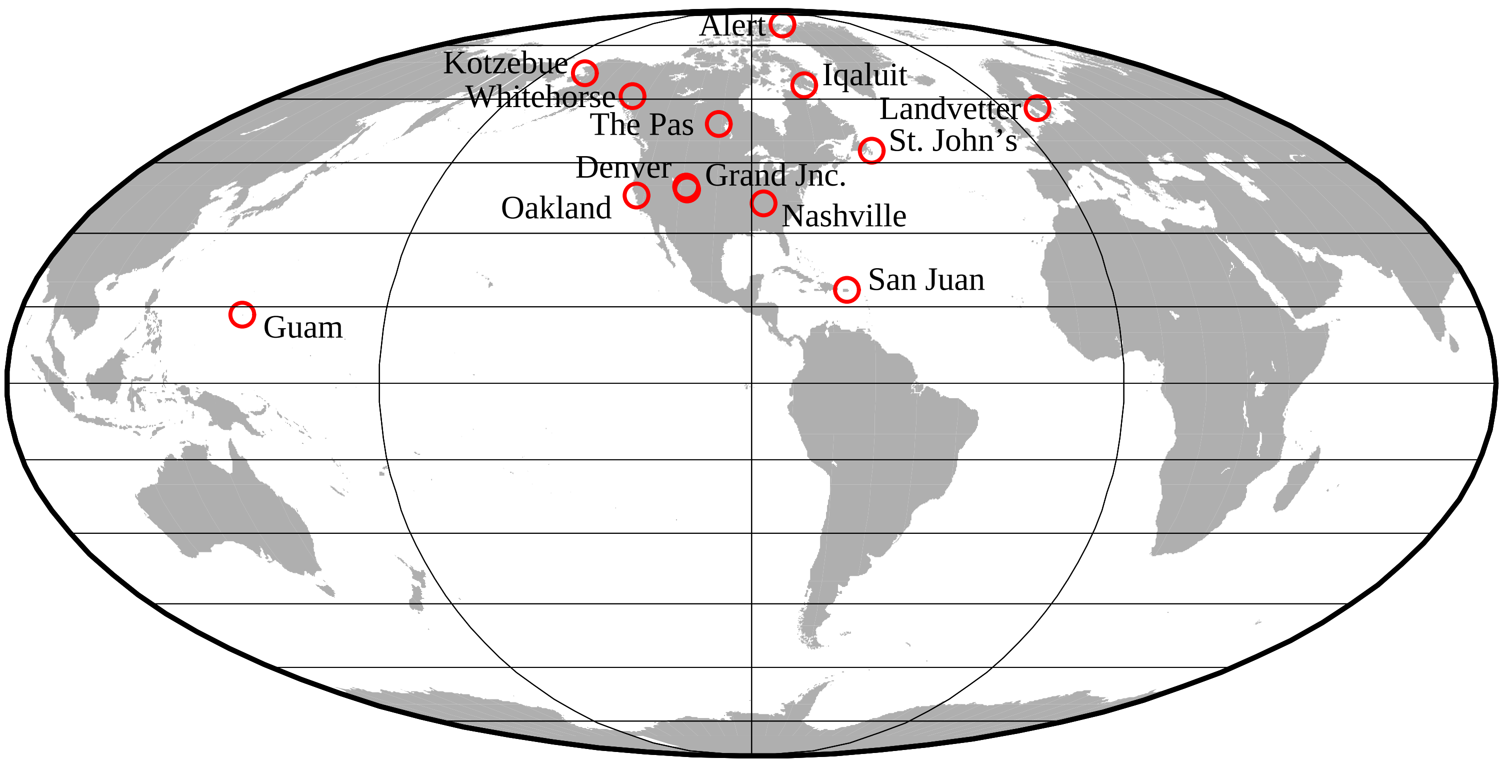
PRINCIPLES OF RAY-TRACING



Application of Snell's Law to a spherically stratified medium.

$$n_{R+h_i} \cdot (R + h_i) \cdot \sin(\phi'_{R+h_i}) = n_{R+h_{i+1}} \cdot (R + h_{i+1}) \cdot \sin(\phi'_{R+h_{i+1}})$$

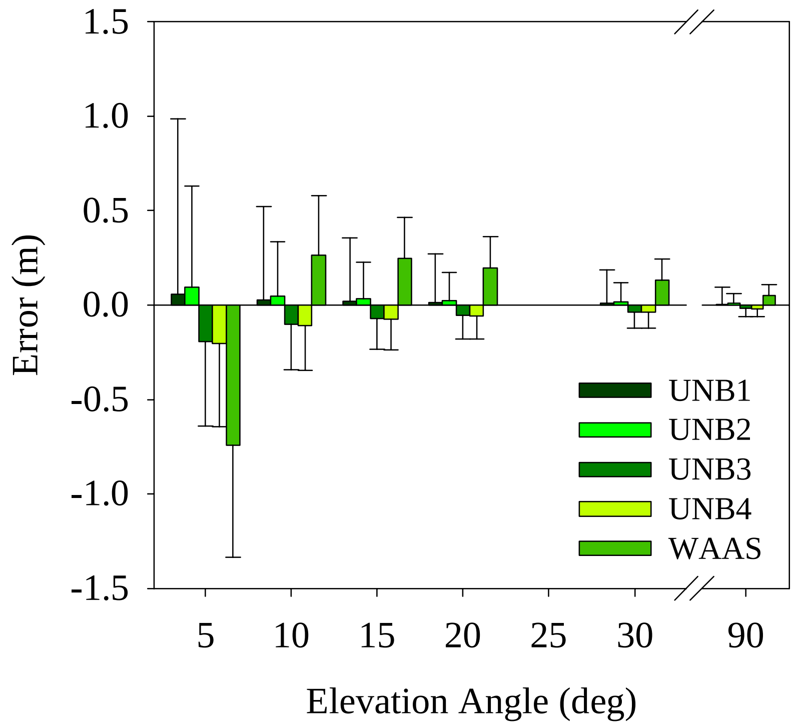
RADIOSONDE LAUNCH SITES



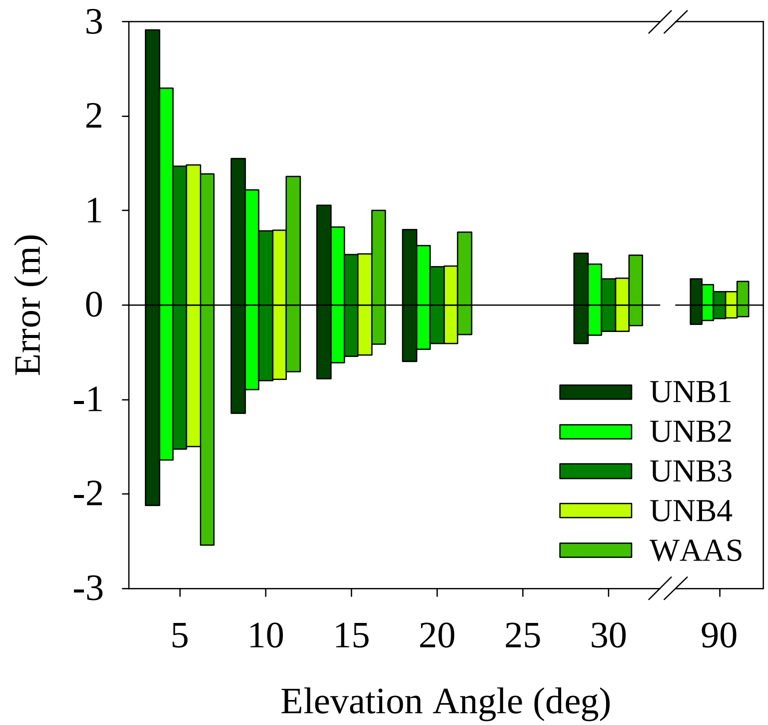
RAY-TRACE RESULTS

(Ray-trace minus model prediction)

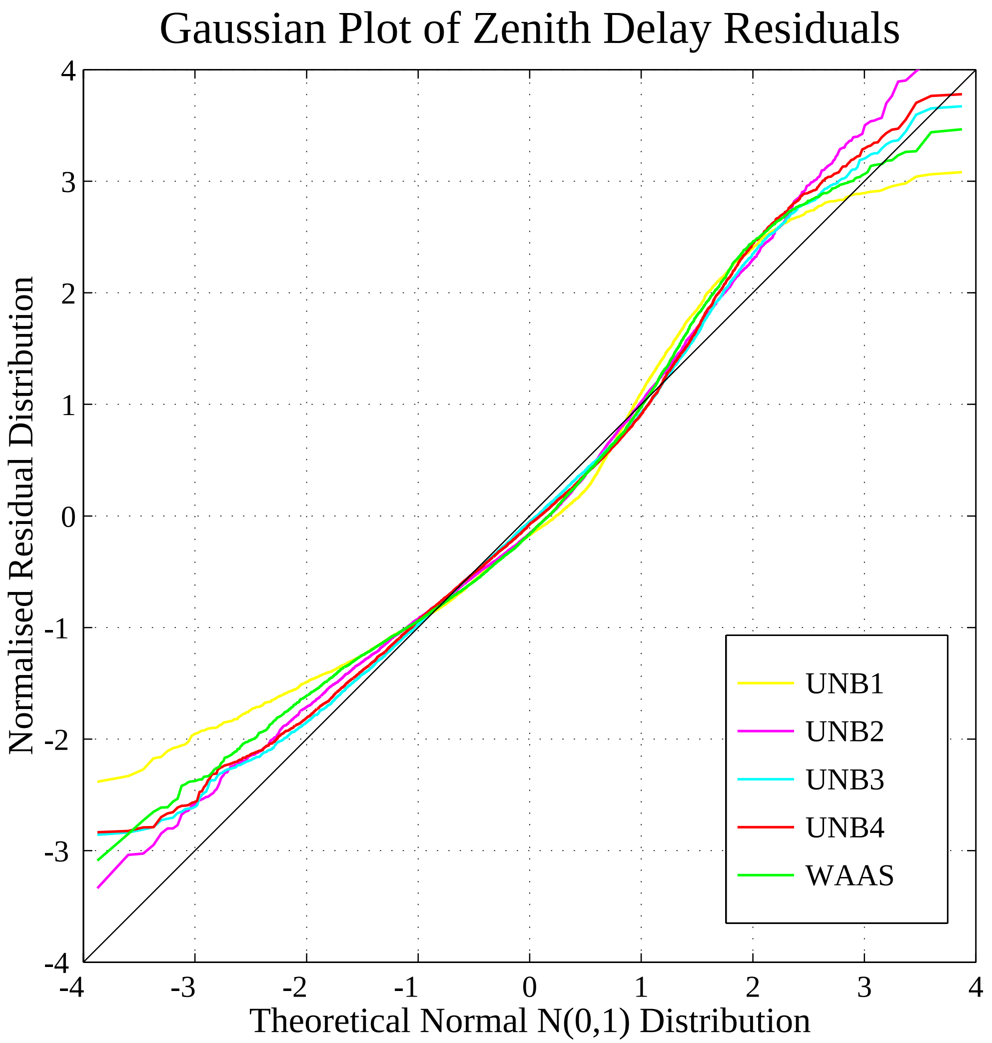
Residual Distribution
Mean and Standard Deviation



Residual Distribution
Maxima and Minima



NORMAL DISTRIBUTION TEST



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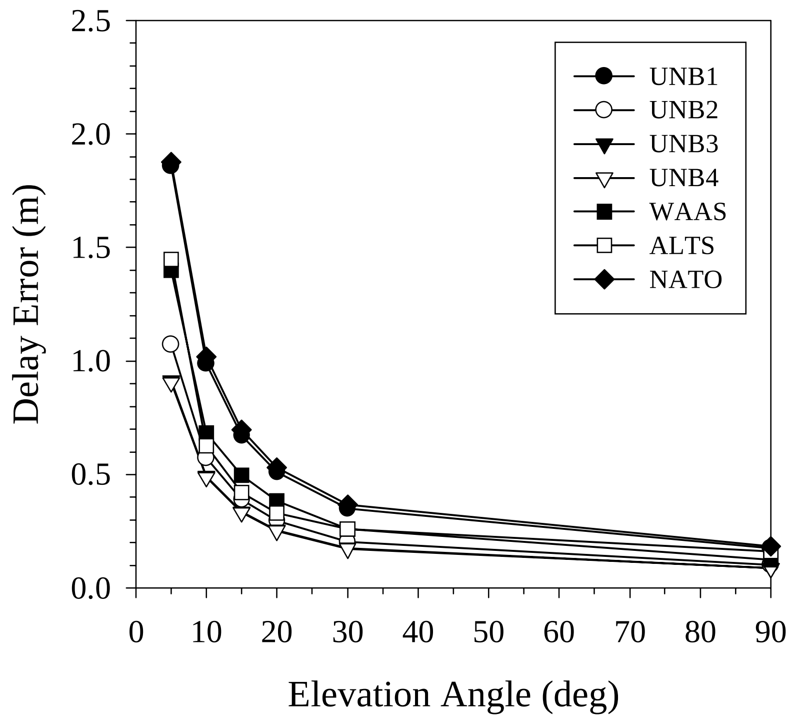
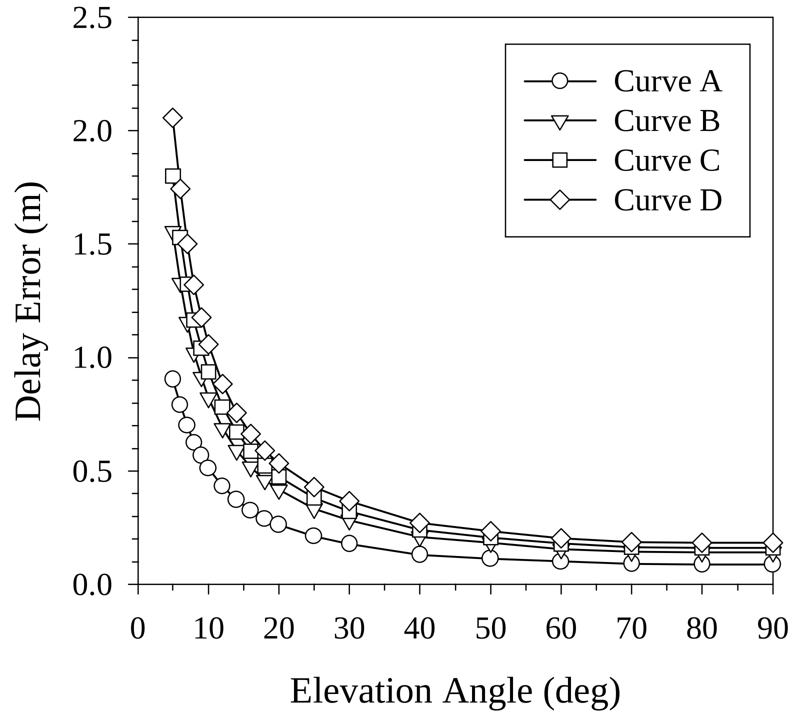
TROPOSPHERIC DELAY ERROR LIMIT

$\overline{x} + 2\sigma$ Range Error
(from ray-trace data)



1σ Range Error
for AFCRL model



EXTREME CONDITIONS

- **Hydrostatic Delay:**
  - ➡ Nominal: 2.30m @ zenith
    23.32m @ 5 deg.
- SLP = 900 mbar, delay = 2.06m
  - ➡ range error = -0.26m @ zenith
    -2.65m @ 5 deg.
  - ➡ Height Bias = -2.40m
- SLP = 1084 mbar, delay = 2.47m
  - ➡ range error = +0.17m @ zenith
    +1.73m @ 5 deg.
  - ➡ Height Bias = +1.57m

- **Wet Delay (UNB3, Tropics):**
  - ➡ Nominal: 0.28m @ zenith
    3.00m @ 5 deg.
- max PW = 80mm, delay = 0.51m
  - ➡ range error = +0.23m @ zenith
    +2.46m @ 5 deg.
  - ➡ Height Bias = +2.24m
- min PW = 0mm, delay = 0m
  - ➡ range error = -0.28m @ zenith
    -3.00m @ 5 deg.
  - ➡ Height Bias = -2.73m

CONCLUSIONS

- Navigation-type models unsuitable for aircraft navigation.
  - ➡ Poor height dependency.
  - ➡ Poor mapping functions.
- Models based on physical principles are superior.
- Provision of “correct” meteorological values important.
- <u>All</u> models susceptible to extreme conditions.
  - ➡ Only real-time measurements will overcome these problems.
  - ➡ Even so, wet delay error can still be large.
- Under “normal” conditions UNB3 and UNB4 limit delay error to ~1.5m at 5 degrees elevation angle.
- UNB3 model provides best results near surface.
- UNB4 consistent at all altitudes represented by GPS data.

FUTURE WORK

- Investigation of environmental extremes
  - ➡ frequency
  - ➡ duration
  - ➡ magnitude
- Assessment of methods to cope
  - ➡ ground monitoring of atmospheric conditions
  - ➡ provision of "real-time" atmospheric parameters
    - barometric pressure — accurate hydrostatic delay
    - temperature — computation of saturation profile - upper limit of wet delay

ACKNOWLEDGEMENTS

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