PRECISE *A POSTERIORI* GEOMETRIC TRACKING OF LOW EARTH ORBITERS WITH GPS

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ABSTRACT

Precise, *a posteriori* orbit determination is required for a wide variety of spaceborne scientific applications. Current Global Positioning System (GPS)-based orbit determination (OD) methods combine spaceborne GPS (SGPS) receiver tracking measurements with the classical orbit determination technique in an optimal estimation process. The research we have carried out investigated the class of low earth orbiters (LEOs) that are at altitudes below approximately 700 km and/or that have complex dynamic behaviour, making precise orbit modelling difficult. Dynamic OD strategies would require presently unattainable high-fidelity force and spacecraft models to ensure precise tracking estimates. To obviate the need for such elaborate yet insufficient models for use with the classical orbit determination process, a completely geometric approach that follows the traditional geodetic relative positioning technique is proposed. This approach combines SGPS receiver data with freely-available data from a network of terrestrial GPS receivers to estimate the SGPS receiver positions over time. Error propagation studies with synthetic data show that decimetre-level precision of position components is possible with only tens of minutes of data processing. Future work includes augmenting the developed software to process actual SGPS data and developing interpolation algorithms to estimate LEO positions between receiver measurements.

INTRODUCTION

Terrestrial tracking of spacecraft by radio and laser techniques provides accurate data to predict orbits and to determine after-the-fact spacecraft trajectories by means of incorporating the measurements in classical orbit determination algorithms. In the last decade, measurement data from Global Positioning System (GPS) receivers aboard spacecraft have been used to augment high-precision *a posteriori* spacecraft tracking. The primary purpose for such tracking estimation lies in vastly differing scientific investigations requiring precise satellite positioning, such as radar altimetry, satellite gradiometry, and atmospheric limb sounding. Also, this fidelity of tracking data has spurred the wider scientific community to heighten its applications-driven positioning requirements.

In this paper, the concept of GPS-based spacecraft tracking is briefly described, along with its measurement benefits compared to conventional tracking techniques, and current processing strategies.

The proposed geometric strategy for GPS-based low earth orbiter (LEO) tracking is then described. A precision assessment of the geometric strategy follows. Finally, conclusions regarding the merits of this proposed tracking strategy are summarised and an outline of further work is described.

PRECISE GPS-BASED LEO TRACKING

Precise spaceborne GPS-based LEO tracking consists of placing at least one SGPS receiver aboard the LEO and processing its carrier phase and/or pseudorange measurements with measurements collected simultaneously at terrestrial GPS observing sites, an approach known as relative positioning. The terrestrial receiver–to–spaceborne receiver measurements are then incorporated into a classical orbit determination estimation algorithm. The mathematical models for the carrier phase and pseudorange observable are:

$$\Phi = \rho + c \cdot (dT - dt) + \lambda \cdot N - d_{ion} + d_{trop} + \varepsilon \quad \text{and} \quad P = \rho + c \cdot (dT - dt) + d_{ion} + d_{trop} + e ,$$

where Φ and P are the measured carrier phase and pseudorange (in distance units) respectively, ρ is the geometric range from receiver to the GPS satellite, c is the vacuum speed of light, dT and dt are the offsets of the receiver and GPS satellite clocks from GPS Time respectively, λ is the carrier wave length, N is the number of cycles by which the initial phase is undetermined, d_{ion} and d_{trop} are the delays due to the ionosphere and the troposphere respectively, and ε and e represent the effects of multipath, receiver noise, and other minor errors on the carrier phase and pseudorange observables respectively. (Note that $\varepsilon \ll e$.) Measurement combination and differencing can almost entirely remove dT (between satellite single difference), dt (between receiver single difference), d_{ion} (dual frequency carrier phase and pseudorange ionosphere-free combinations), and N (carrier phase triple difference); however, combination and differencing increases the noise of the resultant observables and also reduces measurement strength.

The most appealing aspects of the use of GPS measurements are the three-dimensional nature, the precision, and the continuous collection of the measurements. Conventional techniques are limited to those data collection periods when the spacecraft is in line-of-sight of a tracking station. The high costs involved with the operation of these stations and their land-based requirement limits their use and hence reduces tracking data quantity and distribution along the spacecraft orbit. GPS tracking improves upon this situation by providing measurements throughout any orbital arc and requires additional data from only relatively inexpensive terrestrial GPS sites. Also, as opposed to those conventional techniques which provide ground-based range and range-rate measurements resulting in less precise cross track and along track estimates than radial estimates; the three dimensional nature of the GPS measurements allows for superior three-dimensional positioning. Finally, as will be discussed, GPS spacecraft tracking can potentially require substantially less complex estimation procedures than the existing strategies.

PRECISE SPACEBORNE GPS ORBIT DETERMINATION STRATEGIES

Three basic strategies are presently in use to determine precise LEO orbits with GPS. They are the dynamic, the kinematic or non-dynamic, and the hybrid or reduced-dynamic strategies. Each will be briefly described, their accuracies stated, and their weaknesses discussed.

DYNAMIC STRATEGY

In the dynamic strategy, mathematical models of the forces acting on the LEO and mathematical models of the LEO's physical properties (altogether usually referred to as dynamic models) are used to compute a model of the LEO's acceleration over time via the constraints of Newton's second law of motion. Double integration of this model using a nominal spacecraft state vector produces a nominal trajectory – thus developing the equations of motion of the LEO. A model trajectory is then estimated by selecting the LEO state that best fits (*e.g.*, in a least squares sense) the pre-processed (undifferenced or differenced) GPS tracking measurements.

An example of the most accurate SGPS dynamic orbit solution compared to satellite laser ranging (SLR)/Doppler Orbitography and Radiopositioning Integrated by Satellite (DORIS) orbits is for the ~1300 km altitude TOPEX/Poseidon satellite. Using similar dynamic OD processing and dynamic models for both GPS and SLR/DORIS solutions, results of approximately 3 cm, 10 cm, and 9 cm (r.m.s.) for the radial, along track, and cross track orbit components, respectively, were obtained [Schutz *et al.*, 1994]. Ten day arcs comprising double-differenced, ionosphere-free carrier phase and P-code observables were used without the degrading effects of Anti-Spoofing (AS).

This OD strategy also allows for the simultaneous estimation of other parameters to improve the fit between the nominal trajectory and the tracking data, while still preserving available measurement strength by means of the dynamic models. These parameters can be classified as perturbing force and geometric parameters (*e.g.*, gravity coefficients and terrestrial observing station coordinates), and empirical parameters (*e.g.*, once- or twice-per-orbit revolution accelerations) [Yunck, 1996]. Over a long data arc, the effect of noisy instantaneous tracking measurements on the solution are reduced, given that the dynamic models are adequate. However, errors in these models will result in steadily growing systematic errors in the LEO state for longer data arc lengths. For example, empirical parameter estimation indicates weakness in the dynamic models, which generally increases with decreasing LEO altitude, and increasing LEO dynamics.

KINEMATIC STRATEGY

In the kinematic or non-dynamic strategy, the trajectory smoothing caused by the dynamic constraints in the estimation process is removed. The rationale for this is that, particularly at lower altitudes, the actual path of the LEO may be closer to the precise GPS position estimates than the trajectory determined via the dynamics. This strategy can be applied by estimating in a Kalman filter formulation, along with the spacecraft state, a process noise vector representing three force corrections at each measurement epoch. Increasing the process noise can reduce almost completely the effects of the dynamic models. Simulated results for the ~700 km altitude Earth Observing System (EOS) satellite indicate that a radial precision of approximately 3 cm could be achieved with this approach given a data arc of almost one day [Yunck, 1996].

The kinematic OD strategy is therefore actually based on an underlying dynamic formulation, however dynamic modelling errors are circumvented. The strategy relies almost entirely on the precision of the GPS observations and the strength of the observing geometry – that is, the relative location of the LEO and terrestrial receivers with respect to the GPS constellation, and the continuous GPS satellite tracking from the SGPS receiver and the terrestrial GPS receiver array. Until recently, these measurement requirements represented severe problems due to receiver limitations [Melbourne *et al.*, 1994] and insufficient ground arrays [Yunck *et al.*, 1986].

HYBRID STRATEGY

The previous two strategies each have counterbalancing disadvantages: various mis-modelling errors in dynamic OD, and GPS measurement noise in kinematic OD. A hybrid dynamic and kinematic OD strategy would down-weight the errors caused by each strategy, but still utilise the strengths of each. One such strategy has been devised and is referred to as reduced dynamic orbit determination [Wu *et al.*, 1991]. Its basis is again the kinematic correction of the dynamic solution with continuous GPS data. However, by not completely removing the LEO dynamic and spacecraft models, a more accurate solution is possible because sensitivity to mis-modelling and GPS measurement error are both reduced. The weighting of the kinematic and dynamic data is performed again via the Kalman filter process noise. The process noise model contains two primary parameters: a time constant τ_i that defines the correlation in the dynamic model error over one update interval, and the dynamic model steady state variance σ_i^2. When $\tau_i \rightarrow \infty$ and $\sigma_i^2 \rightarrow 0$, the technique reduces to the dynamic strategy, and when $\tau_i \rightarrow 0$ and $\sigma_i^2 \rightarrow \infty$, it approximates the kinematic strategy [Wu *et al.*, 1991].

Orbit determination results using the reduced dynamic technique for the TOPEX/Poseidon satellite have been consistent with results obtained with conventional dynamic techniques using GPS, and SLR and DORIS tracking data. Moreover, using refined dynamic models produces solutions that are even more similar, with differences of only a few centimetres (r.m.s.) in altitude [Melbourne *et al.*, 1994].

In the hybrid strategy, the proper weights of the process noise parameters must be chosen to give the most accurate orbit solution. These values can be derived from computer simulations, covariance analysis, or can be determined from real data. Once the correction parameter values are used, this strategy provides equal or better accuracy compared to the other two strategies.

PROPOSED GEOMETRIC STRATEGY FOR GPS-BASED LEO TRACKING

As discussed, at low altitudes, *e.g.*, below 700 km, the hybrid strategy reduces to the kinematic strategy [Wu *et al.*, 1991], in which the effects of the dynamic smoothing of the GPS measurements by the inadequate dynamic models is removed. The kinematic strategy is required for LEOs that are in such described low orbits and also for LEOs that possess complex orbital motion, such as tethered vehicles or spacecraft of complex dimension and mass distribution (*e.g.*, a space station). However, the usefulness of classical orbit determination in this strategy is greatly reduced by the nearly complete removal of the effects of the dynamic information. Also, orbit determination improvement was initially developed to estimate spacecraft trajectories at varying epochs given discontinuous, imprecise tracking measurements. This is not the case with continuous, precise GPS-based tracking positions. Given these factors, a much less complex and very efficient tracking approach is proposed. The procedure basically uses an augmented form of GPS relative positioning: simultaneous measurements from the LEO receiver and individual receivers at known locations from a terrestrial array are processed to determine the position of the LEO with respect to the terrestrial receivers.

Such a tracking strategy had been advocated in the early development of spacecraft tracking with GPS (*e.g.*, Yunck *et al.* [1986]); however, it was abandoned for dynamic strategies due to the depletion of data strength caused by the large number of parameters required to be estimated. At the time, precise GPS orbits were not available and therefore had to be estimated simultaneously with the LEO position in the solution. This would have required a large global array of terrestrial reference stations that was then unavailable. Given ideal circumstances, decimetre position component precisions were predicted in simulations [Yunck *et al.*, 1986]. If the GPS orbits were not simultaneously estimated, LEO position precision would then be limited to the metre-level.

With the advent of global terrestrial receiver arrays such as the one maintained under the auspices of the International GPS Service (IGS), precise GPS orbits, a large global array of terrestrial GPS reference station data, and associated station coordinate and tropospheric zenith path delay estimates are now available [Neilan *et al.*, 1997; Gendt, 1998]. A great deal of effort and expertise are involved in the generation of these data products, and the geometric strategy represents an opportunity to utilise these data as opposed to either re-estimating some of them or ignoring them altogether. The idea of reducing the computational burden by utilising IGS precise GPS orbits in dynamic OD for example has been tested with only minor reductions in orbit accuracies [Davis *et al.*, 1997].

The inputs to the proposed geometric tracking strategy are precise GPS ephemerides, terrestrial array receiver measurements, receiver coordinate and tropospheric zenith path delay estimates, and dual-frequency pseudorange and carrier phase SGPS receiver data. The measurement data are not fit against a nominal trajectory determined by dynamic models and thus the development of such models is not required. The double-differenced, ionosphere-free pseudorange observable is used to determine noisy (*i.e.*, metre-level) absolute LEO position estimates. The double-differenced, ionosphere-free carrier phase observable differenced between adjacent epochs (triple-difference) is used to determine highly precise (*i.e.*, sub-centimetre-level) LEO position change estimates. This measurement processing technique is a derivative of the process described by, *e.g.*, Yunck and Wu [1986] and Kleusberg [1986], and is a generalised form of carrier-aided pseudorange smoothing. In effect, the low noise carrier phase information is used to map the pseudorange information from all epochs to one epoch for averaging, and this is done for every observation. To summarise, the flow charts of Figure 1 compare and contrast the fundamental constituents of the three presently used tracking strategies with the proposed geometric strategy.

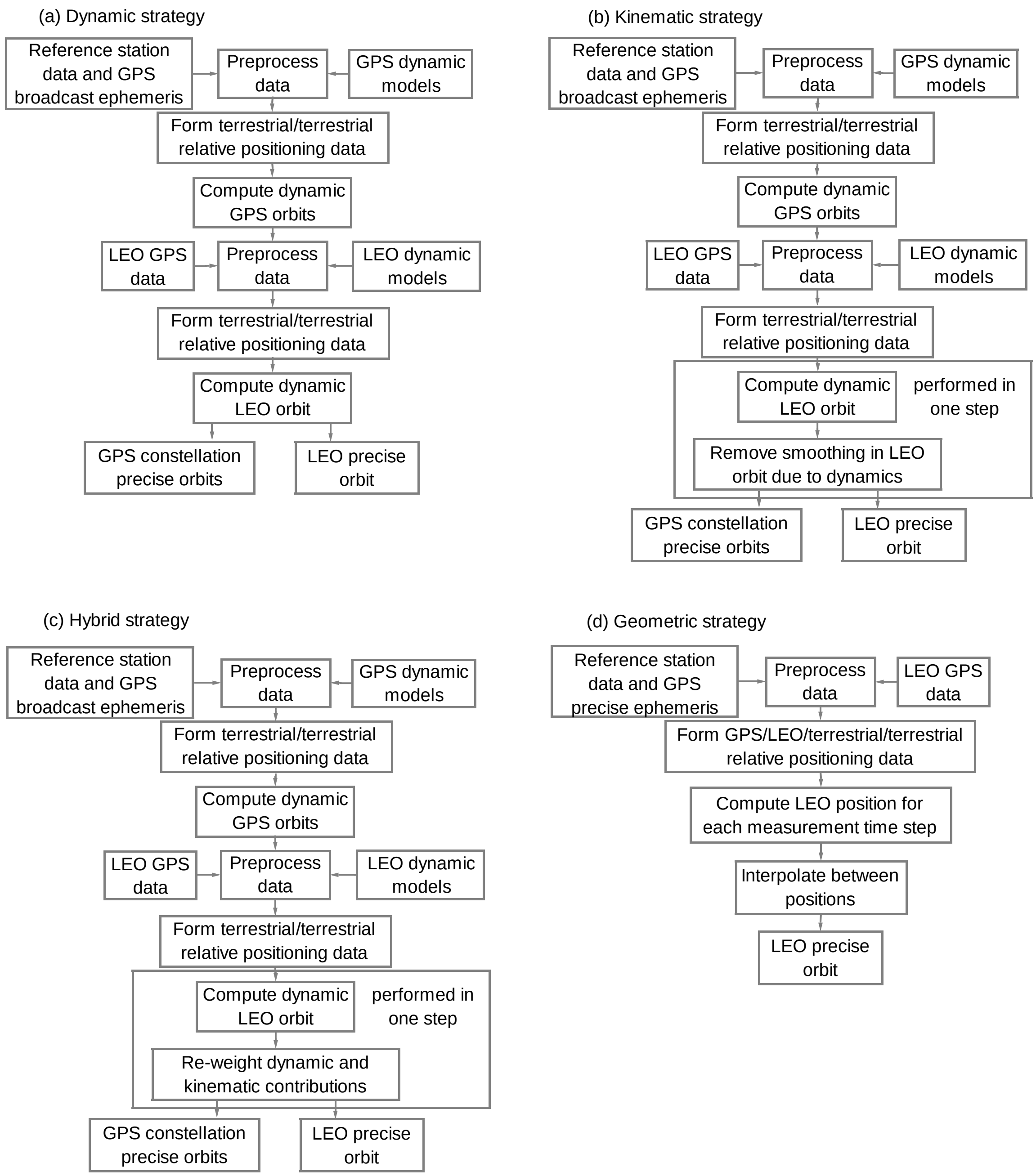

Figure 1. Flow charts of the fundamental constituents of the four GPS-based precise tracking strategies. (a) Dynamic strategy. (b) Kinematic strategy. (c) Hybrid strategy. (d) Geometric strategy.

The theory of least squares is used to solve the processing problem. The general formulation is

$$\mathbf{AX} + \ell = \mathbf{v}; \qquad \mathbf{C}_\ell ,$$

where $\mathbf{X}$ is the vector of unknown parameters, ℓ is the vector of observations, $\mathbf{A}$ is the matrix of measurement partials with respect to the unknowns, $\mathbf{v}$ is the vector of residuals, $\mathbf{C}_\ell$ is the covariance matrix of the measurements. The solution is

$\hat{\mathbf{X}} = \left(\mathbf{A}^T\mathbf{P}_\ell\mathbf{A}\right)^{-1}\mathbf{A}^T\mathbf{P}_\ell\ell$, where $\mathbf{P}_\ell = \sigma_o^2\mathbf{C}_\ell^{-1}$ and $\mathbf{P}_\ell$ is the dimensionless weight matrix and σ_o^2 is the *a priori* variance factor.

The linearised least squares model for our filter is given by:

$$\begin{bmatrix} 0 & \mathbf{A}_{t+1} \\ -\mathbf{A}_t & \mathbf{A}_{t+1} \end{bmatrix}\begin{bmatrix} \boldsymbol{\delta}_t \\ \boldsymbol{\delta}_{t+1} \end{bmatrix} + \begin{bmatrix} \mathbf{w}_P \\ \mathbf{w}_\Phi \end{bmatrix} = \begin{bmatrix} \mathbf{r}_P \\ \mathbf{r}_\Phi \end{bmatrix}; \ \mathbf{C}_P, \ \mathbf{C}_\Phi,$$

where $\boldsymbol{\delta}_t$ and $\boldsymbol{\delta}_{t+1}$ are the corrections to the LEO position at epochs t and t+1 respectively, $\mathbf{A}_t$ and $\mathbf{A}_{t+1}$ are the matrices of measurement partials with respect to the LEO coordinates at epochs t and t+1 respectively, $\mathbf{w}_p$ and $\mathbf{w}_\Phi$ are the differenced pseudorange and carrier phase misclosure vectors respectively, $\mathbf{r}_p$ and $\mathbf{r}_\Phi$ are the differenced pseudorange and carrier phase residual vectors respectively, and $\mathbf{C}_p$ and $\mathbf{C}_\Phi$ are the covariance matrices for the differenced pseudorange and carrier phase measurements respectively. The first matrix equation represents the measurement contribution of the double differenced pseudorange at epoch t+1, and the second presents the change in position between epochs t and t+1 as measured by the triple differenced carrier phase measurement and hence contains position correction parameters for both epochs. The least squares estimate of δx_{t+1} is then used as *a priori* information for the next epoch.

This simple, kinematic, sequential least squares filter allows for the averaging of position estimates over long data arcs, regardless of changing GPS satellites and terrestrial receivers being used in the estimation process. Combining the forward and reverse filter produces a smoothed set of precise point positions of the LEO track. A trajectory can then be fit to this track by means of an interpolation algorithm, *e.g.*, via Legendre or Chebyshev polynomials. The former technique is currently used to re-create precise GPS orbits from a time series of positions [Remondi, 1991]. If however such a procedure is not of sufficient precision, then dynamic model-based orbit fitting would have to be applied. This will be a topic of future work.

Since the proposed geometric strategy relies solely on measurement data, all parameters that affect measurement precision must be taken into account. The first set of factors relates to the measurement capabilities of the SGPS receiver. This includes measurement types observed, measurement noise levels, and the number of available hardware tracking channels. Another LEO hardware issue is the SGPS antenna capabilities: gain pattern, phase centre stability, and field-of-view. Data collection parameters – data arc length and data collection rate – are of great importance. Since this strategy, as with others, is based on relative positioning, the number and distribution of terrestrial receiver sites of precisely known location with respect to the LEO path will also directly affect positioning estimation. Lastly, in relation to the data collection parameters, the associated interpolation algorithm will affect the precision of the trajectory estimation, along with the physical complexity of the actual trajectory. Investigation of some of these items is pursued in the following section.

ASSESSMENT OF GEOMETRIC STRATEGY PRECISION

The following error propagation study was designed to estimate the position determination capabilities of the propose geometric strategy. From the theory of least squares, the covariance of the estimated parameters $\mathbf{C}_{\hat{X}}$ can be determined with only the mathematical and stochastic models $\mathbf{C}_{\hat{X}} = \left(\mathbf{A}^T\mathbf{C}^{-1}\mathbf{A}\right)^{-1}$, which for our model is

$$\mathbf{C}_{\hat{X}_{t+1}} = \begin{bmatrix} \mathbf{A}_t^T\mathbf{C}_\Phi^{-1}\mathbf{A}_t + \mathbf{C}_{\hat{X}_t}^{-1} & -\mathbf{A}_t^T\mathbf{C}_\Phi^{-1}\mathbf{A}_{t+1} \\ -\mathbf{A}_{t+1}^T\mathbf{C}_\Phi^{-1}\mathbf{A}_t & \mathbf{A}_{t+1}^T\left(\mathbf{C}_P^{-1} + \mathbf{C}_\Phi^{-1}\right)\mathbf{A}_{t+1} \end{bmatrix}^{-1},$$

where $\mathbf{C}_{\hat{X}_t}$ is the position estimate from the previous epoch and the position estimate for the current epoch is the upper left sub-matrix of the resulting $\mathbf{C}_{\hat{X}_{t+1}}$ matrix. The elements of measurement partials were computed from laboratory-developed software which computes the GPS, LEO and terrestrial reference station array coordinates and line-of-sight vectors.

The baseline orbit and GPS receiver used for the analysis are from the proposed Canadian BOLAS ionospheric science mission and the Allen Osborne Associates, Inc. TurboStar, respectively. BOLAS stands for Bistatic Observations with Low Altitude Satellites and requires near-decimetre *a posteriori* satellite positions [James, 1997]. The proposed spacecraft would consist of two 70 kg sub-satellites separated by a 100 m tether, rotating in a cartwheel-fashion in its orbital plane. The nominal BOLAS orbital parameters are given in Table 1. The BOLAS GPS requirements would naturally call for the geometric strategy and was in fact a driving force behind the development of the proposed geometric tracking strategy.

Orbital parameter	Value
Perigee	350 km
Apogee	780 km
Period	90 minutes
Inclination	103°
Perigee drift rate	-2.794°/day
Right ascension drift rate	1.657°/day

Table 1. Orbital parameters for the proposed BOLAS spacecraft.

To accommodate the use of GPS on each subsatellite, it has been proposed that a 4π steradian phased antenna array be developed [James, 1997]. In our study, we performed our analyses for both conventional 2π steradian field-of-view (hemispherical) as well as an “all-sky” 4π steradian field-of-view antenna. We did not allow the spacecraft to rotate about its tether and fixed it in a gravity gradient stabilised attitude with the boresight of the 2π steradian antenna pointing to the zenith.

To indicate the geometric strength of the measurements, the position dilution of precision (PDOP) and the number of tracked GPS satellites is given in Figure 2 for a 25 hour period of the BOLAS orbit. The PDOP oscillates about 1.1 for most of the data arc, with a peak-to-peak range of 1.5. The average number of GPS satellites tracked is 10, and at no time is this number less than 5. The geometrical strength can be greatly improved with the 4π steradian LEO antenna, as shown in Figure 3. The PDOP now oscillates about 0.8 and the number of satellites tracked on average increases to an impressive 16, with the minimum being 12.

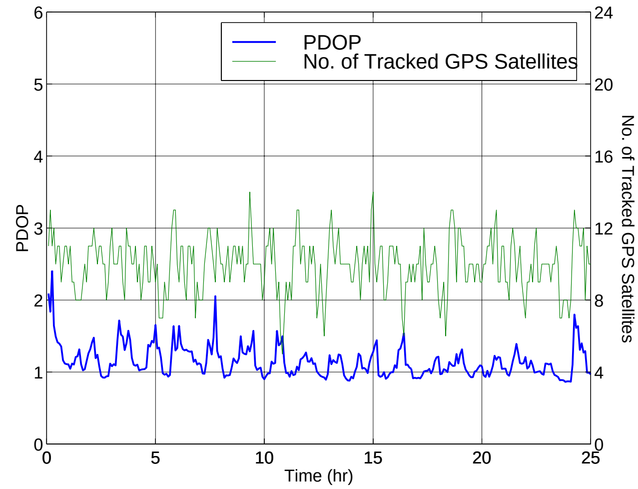

Figure 2: PDOP and number of tracked GPS satellites for 25 hours of simulated data for the LEO with a 2π steradian antenna.

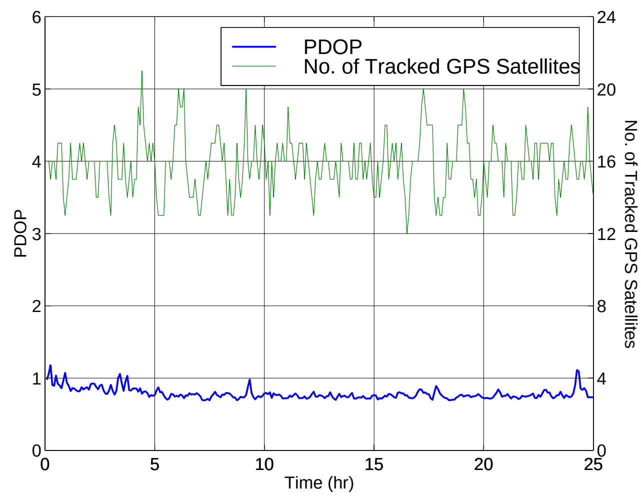

Figure 3: PDOP and number of tracked GPS satellites for 25 hours of simulated data for the LEO with a 4π steradian antenna.

The input noise parameters used in the study are given in Table 2. The SGPS receiver noise is based on the TurboStar receiver [Kunze, 1997]. The pseudorange noise is based on a five minute integration time in the presence of AS. The remaining parameters are defined according to IGS documentation [Neilan *et al.*, 1997; Gendt, 1998]. Note that the ground station zenith troposphere error is mapped to signal elevation angle via the cosecant of the elevation angle. The effects of ionospheric refraction are assumed completely removed by means of the ionosphere-free, dual frequency combination. The effect of signal multipath was not considered.

Parameter	Standard Deviation (cm)
Pseudorange L1	2.2
Pseudorange L2	15.8
Carrier phase L1	0.02
Carrier phase L2	0.3
GPS precise ephemerides	5
Terrestrial GPS station coordinates	1
Tropospheric zenith path delay	1

Table 2: Input noise parameters for error propagation study.

A 25 hour simulated data set was used with a data collection interval of five minutes. The BOLAS ground track for the first 5 hours of this arc is shown in Figure 4. 32 globally distributed ground stations from the IGS network were used, with GPS satellite tracking elevation mask angles of 10º.

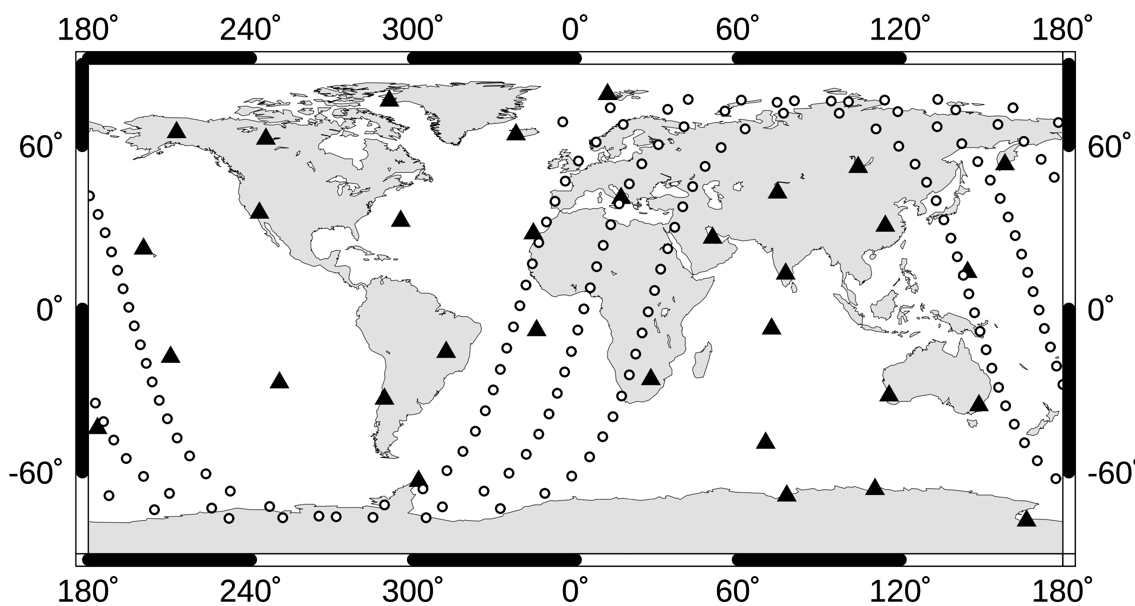

Figure 4: LEO sub-satellite ground track and selected IGS tracking stations for the first 5 hour arc of the error propagation study.

Figure 5 illustrates component position error estimates using a 2π steradian SGPS antenna and Table 3 contains associated summary statistics. The first and last half hour's worth of data have been removed to expel the effects of initial forward and reverse filter convergence. Results show that decimetre-level radial component and sub-decimetre-level along-track and cross-track positioning is possible at the 1-sigma level. The larger mean noise in the radial component is due to the fact that there are no GPS satellites below the LEO – this being comparable to the case for terrestrial GPS receiver use, where the vertical component is always determined with greater uncertainty than the horizontal component. Also of note is that the maximum component error seldom exceeds 15 cm, the total displacement noise – the 3d root sum of squares (3drss) – never exceeds 25 cm, and the standard deviations – noise variations – in each Cartesian noise component is approximately 1 cm.

Turning to the 4π steradian LEO antenna scenario, Figure 6 illustrates the results of the smoother and Table 4 summarises the resulting statistics. The extra measurement strength has reduced the radial and along-track mean noise values, making the radial component error comparable to the errors in the other two components. The repeatability of the estimates across the data arc has also improved with the component noise variations all below 1 cm. A final comment is that the processing time required for each of these scenarios utilising un-optimised, uncompiled Matlab scripts on a 96 MHz MicroSPARC II is approximately 30 minutes, compared to hours with the other processing strategies (see, *e.g.*, Davis *et al.* [1997]).

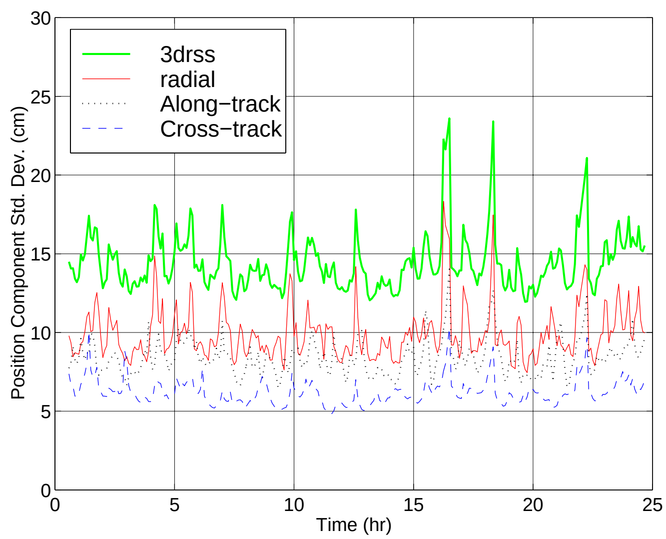

Figure 5: LEO position error estimates for a 24 hour data arc, a 5 minute data collection sampling interval, 32 ground stations, and a 2π steradian antenna.

Component	Max. (cm)	Min. (cm)	Mean (cm)	Std. dev. (cm)
Radial	18.3	7.5	9.9	1.6
Along-track	14.0	6.5	8.4	1.1
Cross-track	10.3	4.8	6.2	0.8
3drss	23.6	11.9	14.4	1.8

Table 3: Summary noise statistics for LEO for a 24 hour data arc, a 5 minute data collection sampling interval, 32 ground stations, and a 2π steradian antenna.

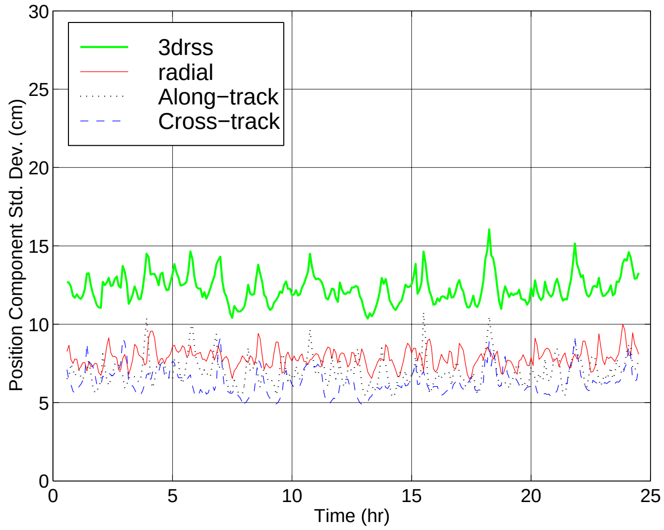

Figure 6: LEO position error estimates for a 24 hour data arc, a 5 minute data collection sampling interval, 32 ground stations, and a 4π steradian antenna.

Component	Max. (cm)	Min. (cm)	Mean (cm)	Std. dev. (cm)
Radial	10.0	6.5	7.9	0.6
Along-track	10.8	5.3	7.0	0.9
Cross-track	9.1	4.9	6.3	0.8
3drss	16.1	10.4	12.3	0.9

Table 4: Summary noise statistics for LEO for a 24 hour data arc, a 5 minute data collection sampling interval, 32 ground stations, and a 4π steradian antenna.

CONCLUSIONS AND FURTHER WORK

Conventional GPS-based tracking strategies exploit the benefits of the precise, continuous, three-dimensional, relatively inexpensive tracking measurements provided by GPS in classical orbit determination estimation, regardless of the level of benefit which is arrived at by smoothing the measurement data. It has been shown that for dynamically complex orbits, the smoothing may produce larger errors than the unaltered measurement-only solution. Also, a wealth of high precision GPS-related data products are being routinely produced that can be used to make GPS-based tracking more efficient. For these reasons, the geometric strategy for GPS-based precise *a posteriori* LEO tracking has been derived.

The strategy involves combining SGPS data with precise GPS ephemerides and terrestrial GPS receiver data in the GPS relative positioning model. Classical orbit determination is not performed, hence there is no need for dynamic models. Furthermore, this strategy allows for precise tracking of LEOs that have complex orbit dynamics. In addition, it maximises the use of existing GPS information (*i.e.*, precise GPS orbits, ground station measurements, and associated data) to reduce processing complexity and cost. We expect that the LEO trajectories can then be interpolated from the position solutions. The overall strategy is very efficient and preliminary error propagation analysis indicates that decimetre-level precision is attainable.

Future work involves improving the GPS filter/smoother, augmenting existing software to process actual SGPS data, developing interpolation procedures, and comparing results with other processing strategies.

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