Pseudorange Multipath Mitigation By Means of Multipath Monitoring and De-Weighting

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BIOGRAPHIES

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ABSTRACT

The management of pseudorange multipath in GPS software processing ranges from total ignorance of the phenomenon to complex schemes for the estimation of the multipath signal. The former can cause significant parameter estimation degradation and the latter cannot necessarily provide accurate estimates. Therefore an alternative method is proposed here, which includes endeavouring to monitor the multipath signal and de-weight the affected observations based on a sentinel observable. The objectives of this approach are the removal of pseudorange multipath-induced position outliers and the reduction of positioning noise.

A modified version of the pseudorange minus carrier-phase observable has been identified in the literature as a possible monitoring observable, and this linear combination is used in our technique. A straightforward, analytical de-weighting function based on this observable is introduced. The relationships between the observable and other weighting criteria such as carrier-to-noise density ratio and satellite elevation angle are also discussed.

Initial static pseudorange testing results indicate that the technique shows great promise, providing 30% to 50% improvement in position estimates in some cases. Similar improvements are shown with carrier-phase smoothed pseudoranges.

INTRODUCTION

The multipath problem has received attention from researchers since the early days of GPS positioning technology. In recent years this interest has peaked due to the great strides that have been made in reducing other components of the GPS error budget. The research performed and reported in this paper was initiated by the need to reduce the effect of this phenomenon on static and kinematic pseudorange-based positioning.

Multipath occurs when signals travelling from a transmitter to a receiver propagate via multiple paths due to reflection and diffraction. The underlying theory of GPS multipath is described in many GPS texts, *e.g.*, Braasch [1996], Kaplan [1996], Langley [1998], *etc.*, and will therefore not be described here. However, multipath compensation strategies will be reviewed, and conceptual explanations will be given where required.

Multipath Mitigation Philosophies

There are four classes of multipath alleviation techniques: judicious antenna siting, hardware solutions, software solutions, and hybrid solutions. All four have advantages, flaws and limitations.

The selection of low-multipath locations for antenna placement is a simple and effective method for reducing

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multipath. This should be done as a matter of course for GPS use. However, it is not always a simple task to predict the level of multipath-induced position error that will be obtained at a particular site. More crucially, it is not always viable to position the antenna in a low-multipath environment.

Hardware compensation rests with antenna design, the use of microwave absorbing material, and receiver tracking augmentation. Extended ground planes and choke rings can reduce antenna susceptibility to ground-bounce multipath, and gain-pattern-forming techniques have been developed to further reduce antenna sensitivity to multipath at low elevation angles. The application of microwave absorbing material on the surfaces close to the antenna also reduces the effect of multipath. Many of the chief advances have also come from receiver-tracking technology. Manufacturers have succeeded in effectively reducing signal tracking correlator spacing, disallowing long delay multipath from being erroneously tracked. However, the vulnerability of these tracking loops to short delay multipath is still of major concern for accurate positioning.

The third category of multipath countermeasure is software mitigation. Algorithms have been developed to attenuate unknown measurement error sources, including multipath, ranging from the application of GPS satellite elevation angle masks to the use of receiver autonomous integrity monitoring (RAIM) schemes (see, *e.g.*, Parkinson and Axelrad [1998]). For static, permanent antennas, the repeat nature of the GPS satellite constellation orbits allows for the estimation and removal of the majority of carrier phase and pseudorange multipath as the identical transmitter-reflector-antenna geometry is observed on approximately a daily basis (see, *e.g.*, Georgiadou and Kleusberg [1988] and Bishop *et al.* [1994]). Fixed multipath geometry can in itself be sufficient to estimate pseudorange multipath (see Kee and Parkinson [1994]). The estimation problem becomes considerably more difficult if neither the recurrence nor the fixed geometry property can be exploited. A number of attempts have been made to estimate pseudorange multipath in kinematic data utilising a Kalman filtering approach (see *e.g.*, de Jong [1999] and Kim and Langley [2000]). Difficulties arise due to low redundancy and the inability to clearly separate the multipath and receiver noise constituents of the pseudorange signal. The last subset of software solutions avoids the estimation of multipath by altering the stochastic model instead (*e.g.*, Wieser and Brunner [2000]). This is accomplished by utilising analytical or empirical weighting functions and subsequently improving these functions by residual analysis of the estimation process.

The last class of multipath amelioration combines hardware and software components to estimate multipath due to the spatial correlation of the measurements received from an array of antennas, but requires the array to be static (see, *e.g.*, Ray *et al.* [1999] and Farret and Santos [2001]).

The software de-weighting philosophy is followed in our research to mitigate the effects of pseudorange multipath. The de-weighting strategy is introduced and its application described. The effectiveness of the approach is characterized in the position solution domain via initial tests and analysis with a number of data sets. We conclude the paper with a summary of our findings and a concise discussion of potential ensuing research.

DE-WEIGHTING TECHNIQUE

The development of this technique is based on the work of Braasch [1994] and others. These researchers observed that a judicious linear combination (the so-called "code-minus-carrier" combination) of the GPS observables produces an observable containing a biased estimate of the pseudorange multipath and a few small additional terms.

Conceptually, in our mitigation approach, the multipath constituent in the pseudorange functional model is not treated as a deterministic quantity to be estimated, but rather it is coupled with the receiver thermal noise and tracking error terms and its variance is estimated with the above linear combination and applied to the stochastic model. Even though this is theoretically inaccurate, it allows for compensation of the effects of the pseudorange multipath in the stochastic model, so long as realistic stochastic models are applied for each epoch in the position estimation process.

Sentinel Observable

The monitoring observable is created as follows. First, the L1 carrier-phase is subtracted from the L1 pseudorange resulting in

$$\begin{aligned} & P_1 - \Phi_1 \\ & = 2\mathrm{dion}_1 - \lambda_1 N_1 + M_1 - m_1 + \mathrm{dTRK}_1 - \mathrm{dtrk}_1 + E_1 - e_1, \end{aligned} \qquad (1)$$

where Φ_1 and P_1 are the measured carrier phase and pseudorange (in distance units), respectively; dion_1 is the delay due to the ionosphere; λ_1 is the carrier wavelength; M_1 and m_1 represent the effect of multipath on the pseudoranges and the carrier-phases, respectively; dTRK_1 and dtrk_1 represent the effect of dynamics-induced tracking error on the pseudoranges and the carrier-phases, respectively; and E_1 and e_1 represent the effect of receiver noise on the pseudoranges and the carrier-phases, respectively. As can be seen, the effects of geometry, receiver and satellite clock errors, and the troposphere are cancelled in this differencing. However, aside from multipath, tracking error, and receiver noise, there are also the dominant components related to the ionospheric delay and the L1 integer ambiguity remaining in the observable. Satellite and receiver hardware delays and other small effects have been ignored as they have negligible effect in this derivation.

The ionospheric delay term can be removed by estimating the dual-frequency biased ionospheric delay from the L1 and L2 carrier phases. Once this term is removed from (1), we are left with the remaining terms and a real-valued phase ambiguity term. To remove the two ambiguity terms, the mean of all of the observable values is subtracted from the observable values. The resulting observable, which we have coined the "pseudo-multipath observable (pm)" is

$$\mathrm{pm}_1 \approx \mathrm{M}_1 - \mathrm{m}_1 + \mathrm{dTRK}_1 - \mathrm{dtrk}_1 + \mathrm{E}_1 - \mathrm{e}_1 . \qquad (2)$$

This quantity is not exact as it contains a small (few centimetre to decimetre-level) residual ambiguity term. Also, it does not represent just the pseudorange multipath, but also the carrier-phase multipath, the pseudorange and carrier-phase tracking errors, and the pseudorange and carrier-phase receiver noise terms as well. The magnitude of the phase terms are minor relative to the pseudorange terms and are therefore ignored. The pseudo-multipath observable is therefore a measure of the pseudorange multipath, tracking error and receiver noise, given that after atmospheric and clock effects are modelled these are the only terms remaining in the pseudorange position estimate. This observable then is a very good indicator of the remaining error (noise and unmodelled terms) in the position estimation model.

Implementation of Pseudo-Multipath Observable

The pseudo-multipath observable in position estimation is used in estimating the pseudo-multipath variance of each satellite tracked for each epoch from continuously evaluating the observable for each satellite and applying these estimates in the positioning filter. For dual-frequency data, the ionosphere-free combination of the observables is used for variance estimation. A simple fixed-interval, moving-variance algorithm is used, in which the r.m.s. is computed. The use of the variance would eliminate any bias over the computing interval and hence is not used. It has been found that the success of the technique is not overly sensitive to the window size selected – a few minute interval has worked well given a 30 second data sampling interval.

Figure 1 depicts the measurement processing flow with the added mitigation routines. The main augmentation is developed in the measurement pre-processor. For each satellite tracked, after the cycle-slip and data gap detection routine is run, the sentinel observable is constructed. Even if pseudorange measurements represent the only observable being processed for position estimation, knowledge of disturbances in the carrier-phase observable is required for pseudo-multipath bias estimation. Once the sentinel observable is constructed, the ionosphere-free transformation of it is computed and the fixed-interval, moving-variance filter is applied. This information is then passed to the main processor for stochastic model construction and filtering to produce solution estimates.

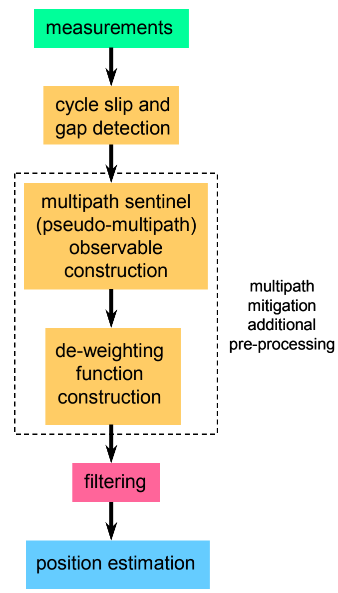

Figure 1: Measurement processing flow augmented by multipath mitigation modules.

Comparison with Other Weighting Functions

Various weighting functions exist for GPS observables. If a stochastic model is used at all, other than an identity matrix, it typically relies on the tracked satellite's elevation angle with respect to the receiver, or on the receiver-computed signal-to-noise ratio (SNR) or carrier-to-noise power density ratio C/N_o. The use of elevation angle-based weighting is very approximate and its use may produce reduced-accuracy positioning results. Transformation equations exist to map receiver C/N_o to noise variance (see, *e.g.*, Langley [1997] and Braasch and van Dierendonck [1999]). However, it may be difficult or not possible to acquire the necessary receiver tracking parameters from the manufacturer to complete the transformations accurately; the equations breakdown at low signal strength levels; and the transformations do not explicitly contain multipath noise, therefore they are not of help in the presented de-weighting technique.

Figures 2 and 3 illustrate these remarks. The data used were collected with a stationary Ashtech Z-12 receiver at Ganong Hall on the University of New Brunswick's Saint John Campus during May of 2000. What appears to be ground-bounce multipath can be clearly seen (quasi-sinusoidal structure) at the beginning of the C/A-code C/N_o time series (b) in Figure 2. The transformation which produces measurement precision (c) does contain again what appears to be a clear multipath signature; however, these values are almost 2 metres less in some cases than the pseudo-multipath precision estimates (e) derived from the pseudo-multipath (d). As is indicated in (e), the pseudo-multipath does a good job in terms of estimating the measurement precision (basically receiver noise in this case) during periods of low multipath levels. The following parameters were used in the transformation from C/N_o to measurement precision: 1 Hz code tracking loop bandwidth (Magellan [2001]), 0.5 early-to-late correlator spacing normalized with respect to one chip (assumed from Langley [1997]), and 0.02 seconds pre-detection integration interval (also assumed from Langley [1997]).

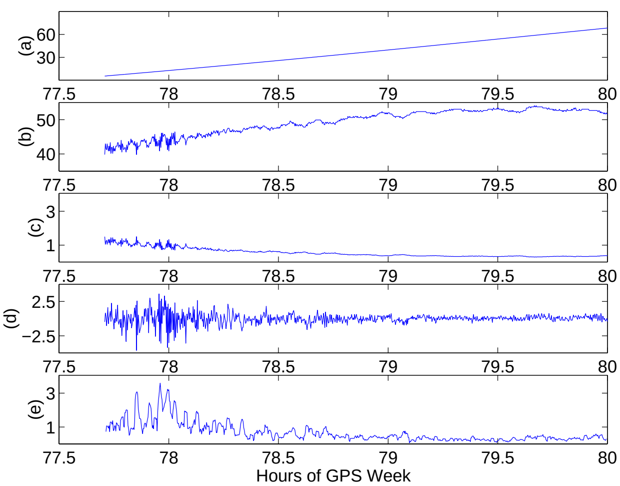

Figure 2: Weighting functions comparison using C/A-code observations collected from space vehicle (SV) 22. (a) SV elevation (deg.). (b) Signal C/N_o (dB-Hz). (c) Pseudorange precision (m). (d) Pseudo-multipath (m). (e) Pseudo-multipath precision (m).

Figure 3 highlights another pitfall of the C/N_o-based noise estimation technique. Even though the SNR values are transformed to reasonable C/N_o values, the transformation to precision is incorrect – the resulting values are much too optimistic. This occurs because the transforming equation is designed for the actual P-code and not the synthesized version which the receiver tracks.

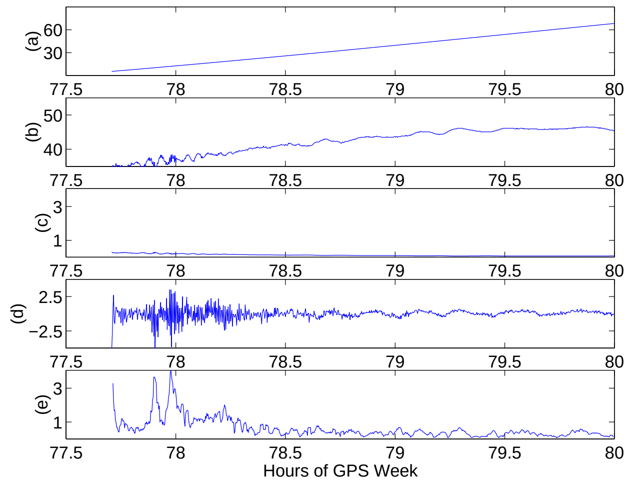

Figure 3: Weighting functions comparison using synthesized P-code observations collected from SV22. (a) SV elevation (deg.). (b) Signal C/N_o (dB-Hz). (c) Pseudorange precision (m). (d) Pseudo-multipath (m). (e) Pseudo-multipath precision (m)

Potential Applications of this Technique

The uses of such a multipath amelioration procedure are quite varied. It could be used for stand-alone static or kinematic receivers, given that the sentinel observables are unaffected by dynamics aside from dynamic tracking error which will map directly into enlarged observable variance. The technique could be used for processing dual-frequency pseudorange data, and it is possible that single-frequency positioning could also be accomplished with broadcast ephemeris-based ionospheric corrections although with less accurate pseudo-multipath estimates. In terms of relative pseudorange positioning, the technique could be used to reduce multipath effects at the remote station, potentially in real-time if bias estimation in the pseudo-multipath observables can be performed accurately. Finally, for applications where pseudorange and carrier-phase observables are combined, such as carrier-smoothing of pseudoranges, this strategy could be used to reduce pseudorange multipath-induced errors. Some of these application areas have been used to test the performance of the de-weighting scheme.

PRELIMINARY TEST RESULTS AND ANALYSIS

Two static data sets were used to test the capabilities of the pseudo-multipath-derived stochastic modelling procedure. Static data were used due to the controlled environment, the clarity of multipath signals in such data, and the constant position solution afford by such data.

The software used for the processing is a point positioning package developed at the University of New

Brunswick (see Bisnath and Langley [2001]). The processor combines ionosphere-free pseudorange and carrier phases (if available) in a kinematic, sequential, least-squares filter. Tropospheric delay is predicted, but residual delay is not estimated at this time. A number of small (in magnitude) geophysical effects also have yet to be modelled in the processor.

Pseudorange Data Testing

The first test uses the Ashtech Z-12 data from Ganong Hall, some of which was shown in Figures 2 and 3. This data set was chosen because, as could be seen in these figures, there exists a significant amount of multipath contamination. Also, there are periods of poor satellite availability, which further complicates the positioning task and magnifies the multipath-induced positioning error. The precise location of the occupied point was not surveyed, so the following analysis is based on epoch position solutions compared to the overall position mean. Therefore no bias information can be gleaned.

Figure 4 shows the position component errors from the pseudorange point positioning solution. Very large divergences exist, with peak-to-peak errors in the height component reaching a staggering 60 metres. The reason for these substantial errors is presented in Figures 5 and 6.

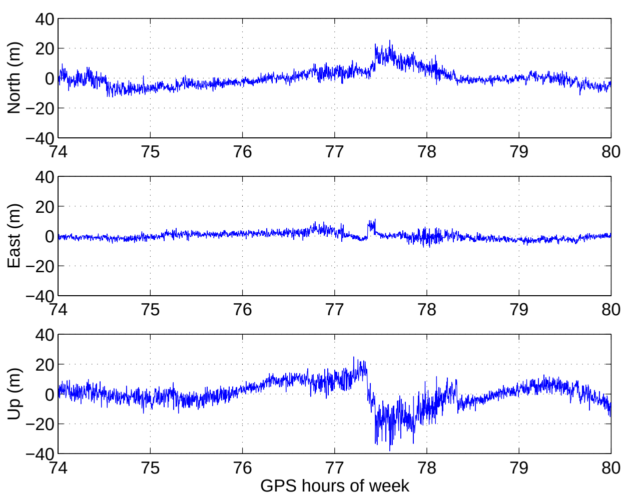

Figure 4: Pseudorange point positioning component differences from the mean for Ganong Hall data set.

Our analysis focussed on the period of the principal error: the few minute interval at about 77.4 hours. At this time three satellites set, the last one being SV11, and only SV06 has risen (see Figure 5). Consequently, the position dilution of precision (PDOP) increased from 2 to 4. Greatly exacerbating this situation are the facts that the degrees of freedom in the estimation procedure has been reduced to 1, and the low elevation satellite data contain significant multipath components as can be seen in Figure 6. This figure shows the high noise levels of these low elevation angle portions of data, reaching an apex of 6 metres as compared to low pseudo-multipath noise in this data set of 2 metres or lower.

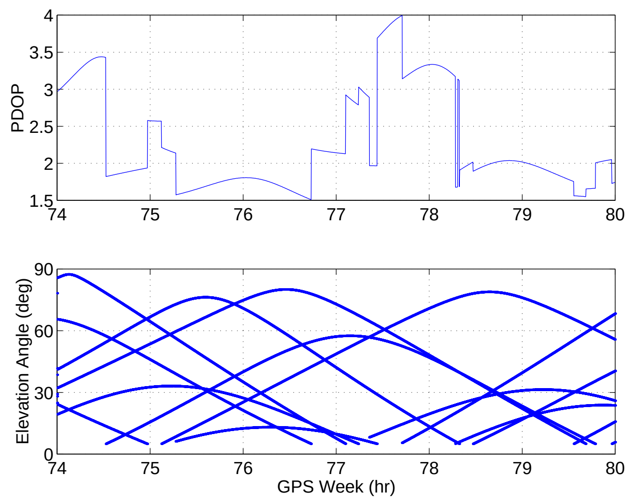

Figure 5: PDOP and elevation angle for all SVs above 5° for Ganong Hall data set.

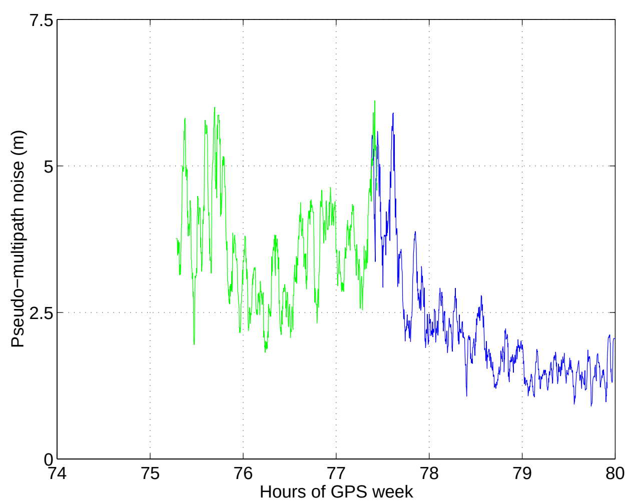

Figure 6: Estimated pseudo-multipath noise for SV11 (light green) and SV06 (dark blue) for Ganong Hall data set.

By applying the pseudo-multipath de-weighting technique, these large divergences are curtailed, as is illustrated in Figure 7. For periods of relatively low multipath, the original errors were not large and were not significantly affected by the de-weighting. The comparative standard deviations for both sets of results are given in Table 1. The percent reduction in this statistic's value is as much as 40% for the vertical error component.

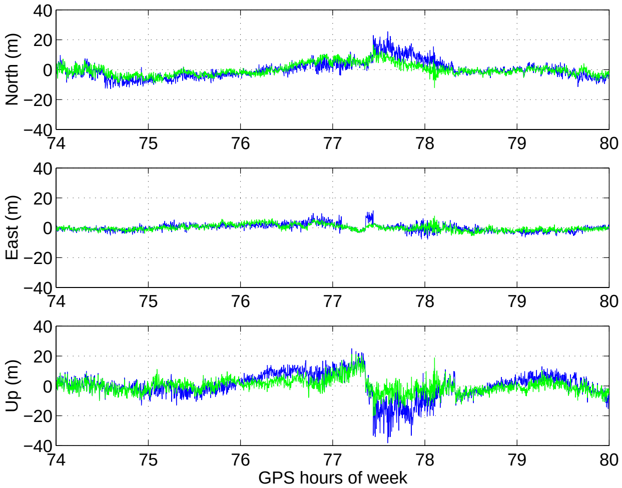

Figure 7: Un-weighted (dark blue) and de-weighted (light green) pseudorange point positioning component differences from the mean for Ganong Hall data set.

Statistic	De-weighting	North	East	Up	3D
std. dev.	No	5.7	2.5	8.2	10.3
	Yes	4.1	2.0	4.8	6.6
	improvement	**28**	**20**	**42**	**36**

Table 1: Statistical summary (in metres) of Ganong Hall pseudorange point positioning. Improvements are in units of percent.

The effect of the de-weighting can also be clearly seen in the pseudorange residuals. Figure 8a shows the original, un-weighted processing residuals for SV11 and SV06 whose pseudo-multipath noise were depicted in Figure 6. The residuals are relatively small for this data set, indicating that even though the multipath constituent of these measurements is high, the measurements play a significant role in the position estimation. This situation is rectified to a large degree with the de-weighted solution and can be seen in the associated satellite residuals in Figure 8b. The residuals have increased from the few metre-level to the 15 metre-level. The rising of SV06 is nicely shown in the reduction of the variance and magnitude of its residuals.

The second data set processed for this analysis was from the Algonquin (ALGO) station of the Canadian Active Control System (CACS) network. The data were collected in August 2000, with a TurboRogue receiver. Being a permanent reference point, the multipath environment is much more benign than is the case for the Ganong Hall data set, and the few millimetre-level precision International Terrestrial Reference Frame (ITRF) coordinates are available for bias and r.m.s. analysis.

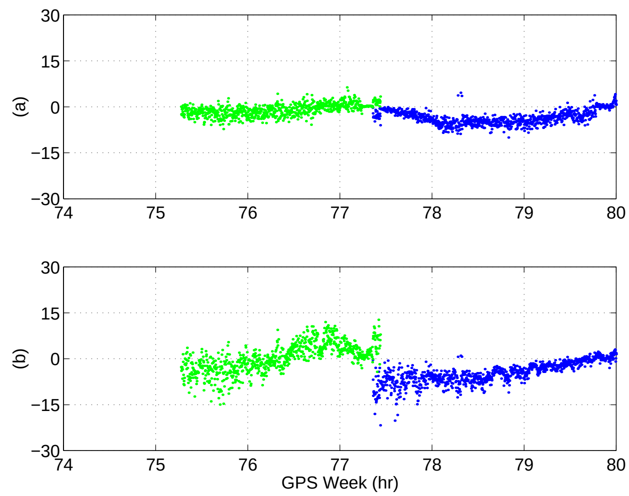

Figure 8: Pseudorange residuals (in metres) for SV11 (light green) and SV06 (dark blue) from un-weighted solution (a) and de-weighted solution (b) for Ganong Hall data set.

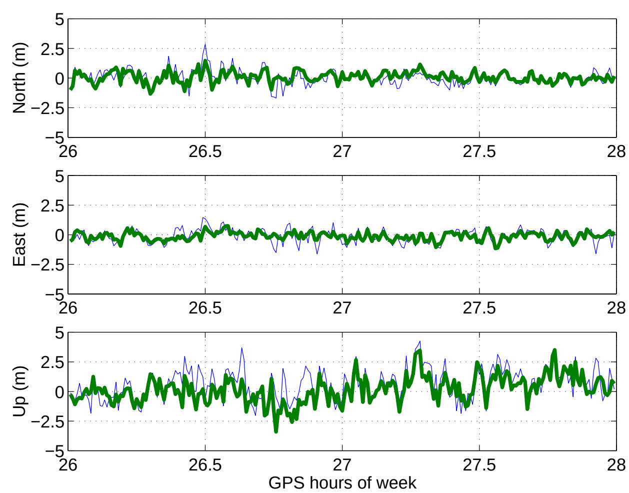

Figure 9: Un-weighted (thin blue line) and de-weighted (thick green line) pseudorange point positioning component errors for Algonquin data set.

Figure 9 shows the component error, with respect to the ITRF position, of the un-weighted and de-weighted pseudorange point positioning solution. It is apparent that the de-weighting provides some improvement, particularly for some of the larger height divergences. The reduction of the standard deviation is about 20% for the total displacement. However, the truly significant improvement comes in the reduction of the bias – from 62 cm to 23 cm total displacement or more than 60% (see Table 2). The majority of the reduction is derived from the vertical estimation improvement.

Statistic	De-weighting	North	East	Up	3D
std. dev.	No	63	56	130	155
	Yes	46	34	112	126
	improvement	27	39	14	19
bias	No	-3	14	-61	62
	Yes	-8	12	-17	23
	improvement	--	14	72	63

Table 2: Statistical summary (in centimetres) of station Algonquin pseudorange point positioning. Improvements are in units of percent.

Pseudorange and Carrier-Phase Data Testing

Another processing strategy which this multipath de-weighting technique is designed to aid is carrier-smoothed pseudorange processing. The Algonquin data set is again used in this testing. A number of improvements are expected with the enlisting of the de-weighting function. These include reduction in the initial position error, reduction in any multipath-induced solution divergence, and of course increased overall positional accuracy.

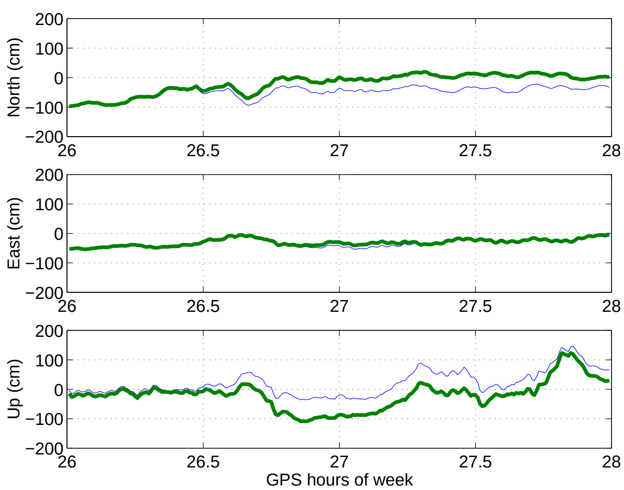

Figure 10: Un-weighted (thin blue line) and de-weighted (thick green line) pseudorange and carrier-phase forward-filter point positioning component errors for Algonquin data set.

Figure 10 shows the component errors in position for the forward run of the point-positioning filter, without and with de-weighting. The initial bias which exists for the north and up components is not corrected. This discrepancy must still be investigated. The convergence of the solution is improved with the de-weighting, particularly in the north component.

The backward filtering (see Figure 11) performs closer to expectation. Initial biases are reduced from 1 or 2 metres to the sub-metre or metre-level, respectively. This results in faster convergence of the filter. Also, a small amount of divergence is removed from the height component between 26 and 27 hours.

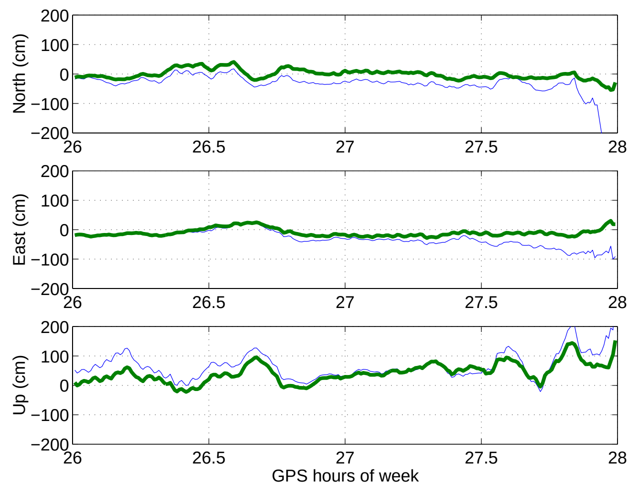

Figure 11: Un-weighted (thin blue line) and de-weighted (thick green line) pseudorange and carrier-phase backward-filter point positioning component errors for Algonquin data set.

Figure 12 shows the smoothed solution resulting from the combination of the forward filter and backward filter runs. As can be seen, the de-weighting increases the positional accuracy of the solution. Table 3 shows the summary statistics for the period starting at 26.5 hours and ending at 27.5 hours. The component r.m.s. improvements with the de-weighting range from 40 to 70 percent, reducing the r.m.s. to 14 cm, 18 cm, and 28 cm in the north, east, and up components, respectively. We believe that this result is quite good given that the processing was performed in a kinematic mode (*i.e.*, no knowledge of the receiver's stationary nature was used), and as previously stated, a number of small error sources have not been accounted for in the processing, including the residual tropospheric delay.

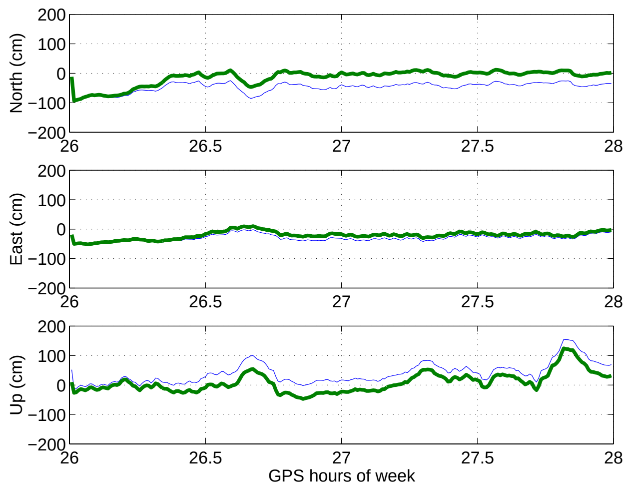

Figure 12: Un-weighted (thin blue line) and de-weighted (thick green line) pseudorange and carrier-phase smoothed point positioning component errors for Algonquin data set.

Statistic	De-weighting	North	East	Up	3D
std. dev.	No	12	11	26	31
	Yes	13	10	28	32
	improvement	**--**	**9**	**--**	**--**
bias	No	45	28	-39	66
	Yes	5	15	-1	16
	improvement	**89**	**46**	**97**	**76**
r.m.s.	No	47	30	47	73
	Yes	14	18	28	36
	improvement	**70**	**40**	**40**	**51**

Table 3: Statistical summary (in centimetres) of station Algonquin pseudorange and carrier-phase smoothed point positioning between hours 26.5 and 27.5. Improvements are in units of percent.

CONCLUSIONS AND RECOMMENDATIONS

The pseudo-multipath observable – a variant of the code-minus-carrier observable, has been used to estimate the collective multipath, dynamic tracking error, and receiver thermal noise error in GPS pseudorange measurements. The use of this observable has been advanced in the presented research to estimate the combined multipath, tracking error, and thermal noise variance. This variance was used to de-weight pseudorange observations contaminated by predominantly multipath in the position estimation process. Initial testing of the technique indicates that 30 to 50 percent reduction in total position displacement error can be achieved with the data sets used. More work is needed to understand fully the behaviour of the pseudo-multipath observable, specifically with kinematic data. Also more analysis is required in the estimation of position de-weighting variance from the pseudo-multipath observable.

We believe that there is a place for this technique in the broader context of quality control. Quality control here is meant to represent among other components, multipath mitigation, stochastic modelling, residual analysis, and outlier detection and removal. It has been shown here that the technique can be used for multipath mitigation. The de-weighting implementation is effectively an improved stochastic modelling technique. The enhancement of the technique can take the form of residual analysis to improve the accuracy of the stochastic modelling. This is possible due to the reasonable stochastic estimates provided by the pseudo-multipath variance. Data outlier detection and removal can also be performed with residual analysis and potentially with the pseudo-multipath observable itself.

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