Satellite and Receiver Phase Bias Calibration for Undifferenced Ambiguity Resolution

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ABSTRACT

Precise point positioning (PPP), considered an alternative to differential positioning, is used in a significantly increased number of applications. Its integration into many practical areas is, however, slowed down by the long convergence time required in order to obtain cm-level accuracy. This drawback is caused by the difficulty in fixing carrier-phase ambiguities to integers. As opposed to the differential mode, where many error sources are eliminated or greatly reduced, PPP has to properly account for all of them. Some of these error sources, such as code and phase biases, are complex to model as they tend to merge with the ambiguity parameters during the estimation process, leading to unsuccessful ambiguity resolution.

This paper focuses on the receiver and satellite phase bias calibration required to recover the integer nature of carrier-phase ambiguities. A proper estimation of these biases would allow correcting the measurements and using the ambiguity resolution techniques developed for differential positioning. In this way, instantaneous cm-level accuracy could be something conceivable even with a single GPS receiver, considering that the other error sources have been reduced to a significant level.

The first step taken to achieve this objective is to clearly understand PPP's functional model representing the code and phase measurements made by the GPS receiver. Special attention is paid to hardware delays, such as code and phase biases, which play a crucial role in the estimation process using undifferenced measurements. The impact of these quantities on some estimated parameters is described in order to have a better understanding of the concepts presented throughout this paper.

In the second step, a receiver phase-bias calibration technique using a GPS signal simulator is introduced. A simulator has been used to generate errorless signals which are ideal to isolate the biases inherent to the receiver. Results show that this calibration process is complex due mainly to the correlation between the

receiver clock and the ambiguity parameters. As such, between-satellite single differencing still seems to be the best way to eliminate receiver phase biases.

The last part of this paper concerns satellite phase-bias calibration. To properly calibrate the satellite phase biases, the impact of code biases has to be carefully taken into consideration. For this purpose, an alternate widelane phase-bias calibration method is proposed and is shown to be coherent with PPP's functional model.

INTRODUCTION

While cm-level positioning accuracy with a single GPS receiver once seemed like a hardly achievable task, the objective is now finding the quickest method to reach this threshold. This problem is of a lesser concern in differential positioning where the phase ambiguity parameters can be treated as integer values. By constraining those parameters to integers, one can usually obtain high accuracy within seconds to minutes depending mainly on the baseline length.

On the other hand, in precise point positioning (PPP), fixing ambiguity parameters to integers is a much more complex task. There are two major concerns to consider when addressing this issue: 1) the error budget affecting the observations must be kept to a reasonably low level (usually within quarter of a carrier wavelength), and 2) the hardware biases affecting the observations need to be adequately handled. The former concern has been investigated by several authors and is beyond the scope of this paper. Instead, a light will be shone on how the phase and code biases become such a nuisance to ambiguity resolution in PPP.

This paper focuses on three aspects aiming at recovering the integer nature of phase ambiguity parameters in PPP. First, the functional model describing the GPS measurements is briefly recalled in order to demonstrate the propagation of unmodeled hardware biases into the estimation process. Following this discussion, a receiver phase-bias calibration method based on the use of a GPS signal simulator is presented. Finally, while keeping in mind PPP's functional model, an alternative to existing methods of satellite phase-bias calibration is introduced.

ANALYSIS OF PPP'S FUNCTIONAL MODEL

Observation Equations

Code observations are mandatory in PPP due to a linear dependency relating the receiver clock and the ambiguity parameters. Hence, PPP's functional model can be described by the following equations:

$$\Phi_i = \rho_i + c\left(dT - dt\right) + T - I_i + \lambda_i \bar{N}_i + w_i + \varepsilon_{\Phi_i} \qquad (1)$$

$$P_i = \rho_i + c\left(dT - dt\right) + T + I_i + b_{P_i} + b^{P_i} + \varepsilon_{P_i} \qquad (2)$$

where

$$\bar{N}_i = N_i + b_{\phi_i} + b^{\phi_i} \qquad (3)$$

and

i	identifies the frequency-dependant terms
Φ_i	is the carrier-phase measurement (m)
P_i	is the code measurement (m)
ρ	is the instantaneous range between the phase center of the satellite and receiver's antennas including earth tides, ocean loading and relativistic effects (m)
c	is the vacuum speed of light (m/s)
dT	is the satellite clock bias (s)
dt	is the receiver clock bias (s)
T	is the tropospheric delay (m)
I_i	is the ionospheric delay (m)
λ_i	is the wavelength of the carrier (m)
N_i	is the integer carrier phase ambiguity (m)
w_i	is the phase windup effect (m)
b_{ϕ_i}	is the receiver carrier phase bias (cy)
b^{ϕ_i}	is the satellite carrier phase bias (cy)
b_{P_i}	is the receiver code bias (m)
b^{P_i}	is the satellite code bias (m)
$\varepsilon_{\Phi_i}, \varepsilon_{P_i}$	are the measurement noise components, including multipath (m)

Impact of Satellite and Receiver Hardware Biases

When combining code and phase observations in point positioning, hardware biases (other than clock biases) become a major concern. Each satellite contains an oscillator having a fundamental frequency (f_0) of 10.23 MHz, used to generate the carriers and the modulations [IS-GPS-200D, 2004]. When combining those components together, several delays can occur, as illustrated in Figure 1. A similar phenomenon can also be observed in the receiver when it generates the signal replica.

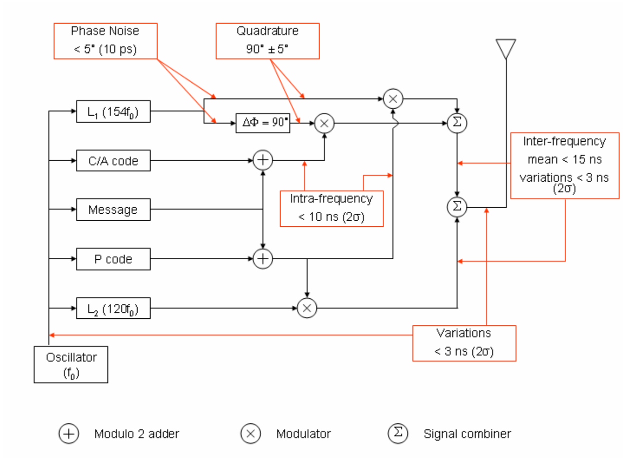

Figure 1: Satellite hardware biases (based on Wells et al. [1987] and IS-GPS-200D [2004])

While different techniques have been developed to estimate the intra-frequency and inter-frequency differential delays of signal paths (see, for example, Schaer [1999] and Gao et al. [2001]), the absolute delay associated with a particular signal or modulation is much more complex to determine. This is due to the linearity of many parameters in Equations (1) and (2), such as the hardware biases, the clock offsets, and the ambiguity parameters.

In the context of GPS data processing, the biases that cannot be eliminated or modeled usually tend to merge with other parameters, thus altering the estimated values. Here is a summary of the way each bias affects the estimation process:

- **Satellite code biases** are mostly eliminated from the code observations by using the satellite clock corrections (from the broadcast message or the International GNSS Service (IGS)) along with the appropriate differential code delay corrections [Collins et al., 2005]. On the other hand, code biases are introduced in phase observations when using satellite clock corrections (refer to the later section "Satellite Phase Bias Calibration").

- The receiver clock parameter absorbs the common part of **receiver code biases** and the non common part is expected to propagate into the code residuals and estimates of other parameters such as the receiver coordinates.

- **Satellite phase biases** are different for each satellite on each carrier frequency and they tend to merge into the ambiguity parameters. This is not a problem when using the ionosphere-free combination because the ambiguities are no longer integers. For ambiguity resolution, this aspect becomes a major concern.

- **Receiver phase biases** are expected to be the same for each satellite but dependant on frequency. They will then merge into several parameters, such as the receiver clock, the phase ambiguities and potentially coordinate estimates.

The abovementioned facts will be used throughout this paper to support the development of the calibration methods. Remember that accurate knowledge of these delays would allow the correction of the phase and code observations in order to obtain not only unbiased receiver time estimation but integer ambiguity parameters as well. For this purpose, the following sections present methodologies for phase bias calibration.

RECEIVER PHASE BIAS CALIBRATION

Until now, few attempts have been made to calibrate the receiver phase biases since they are subject to important variations that are due mainly to the instability of the receiver's oscillator. A zero-baseline test already confirmed that a receiver restart changes the value of the biases [Wang and Gao, 2007], which makes calibration an extremely complex process.

On the other hand, receiver bias calibration offers the advantage of controlling the environment in which the tests are performed. For instance, using a GPS signal simulator allows generating (almost) errorless signals, free from satellite biases. From this perspective, a new calibration method has been investigated to learn about the behavior and the characteristics of receiver hardware delays.

Methodology

In the process of isolating receiver phase biases, a GPS signal simulator has been used to generate phase and code observations free from the following error sources: ephemeris, satellite clock offsets and hardware delays, troposphere, ionosphere, earth tides, ocean loading, phase windup, multipath and antenna phase-center variations. This scenario can be described by simplifying Equations (1) and (2), that is:

$$\Phi_i = \rho + cdT + \lambda_i\left(N_i + b_{\phi_i}\right) + \varepsilon_{\Phi_i} \quad (4)$$

$$P_i = \rho + cdT + b_{P_i} + \varepsilon_{P_i} \quad (5)$$

The satellite and station coordinates being known, the only unknown parameters are the receiver clock offset, the ambiguities and the receiver's code and phase biases. Furthermore, the noise level is greatly reduced in this scenario and depends primarily on signal resolution. For the sake of simplicity, hardware simulator delays have been omitted.

To obtain the receiver phase biases, the ambiguity parameters are estimated as real numbers at every epoch using a least-squares adjustment technique. The fractional part of the resulting ambiguity values is simply considered to be the bias sought. Also, note that the code biases will be estimated as an intrinsic part of the clock offset (refer to the “Preliminary Discussions” subsection hereafter).

Test Description

In order to verify the validity of the proposed methodology, a test has been performed using the Spirent STR4760 GPS signal simulator at the University of New Brunswick and a NovAtel ProPack V3 receiver (Figure 2).

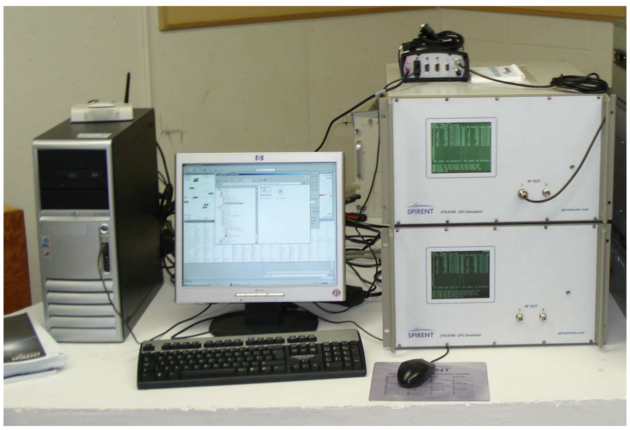

Figure 2: Receiver phase-bias calibration setup

Two sessions lasting approximately three hours each and using the same satellite configuration have been performed. Between each session, the receiver and the simulator have been turned off to observe the behavior of the biases in the context of a receiver reset.

The receiver used outputs phase measurements on L_1 and L_2 as well as the C/A and P_2 code measurements. Since the P_2 code resolution is superior to the one from the C/A code, only the former has been used to compute the clock bias in all cases (except where indicated below).

Preliminary Discussions

Before going any further, an important discussion is required. Even though this methodology has several advantages, it also contains some important drawbacks. As mentioned previously, it is impossible to estimate independently the receiver clock offset and the receiver code biases because they are both linear terms and they affect identically all simultaneous code observations of a given type. The receiver clock parameter will then absorb a great part of the receiver code biases. This has a direct consequence on the values of the ambiguities estimated because of the strong correlation between the receiver clock and phase ambiguity parameters.

To highlight this effect, a simple test has been carried out. Using the methodology described above (Equations (4) and (5)), two scenarios were performed subsequently: first, the unknown parameters (dT and N) have been estimated using L_1 phase observations along with P_2 code observations. Then, the same phase observations were used along with the C/A code instead of the P_2 code.

Figure 3 presents the receiver clock parameter estimated in both scenarios, as well as the corresponding ambiguities estimated independently at each epoch for satellite PRN 1. A different clock bias value can be observed in each case, which is a direct consequence of the differential code bias between both code observations. Also, note that the ambiguity values obtained are different.

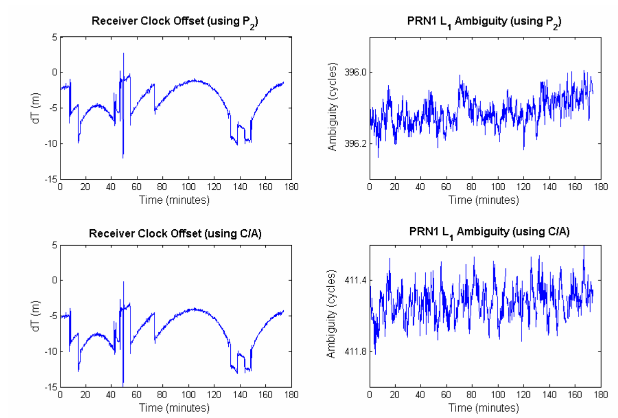

Figure 3: Propagation of receiver code biases in ambiguity parameters

This shows that the receiver code biases alter the estimation of the ambiguity parameters, resulting in an inadequate phase bias.

The ideal solution to this problem would be to calibrate independently the code biases. This issue has already been tackled for time transfer purposes [Petit et al., 2001], but the approximated error budget is still around 1 ns (≈ 30 cm) [Plumb et al., 2005], which is too large for our purpose (it is even greater than the wavelength of the signals).

Results

Even with the constraint previously mentioned, some information can be deduced on the variability of the phase/code biases. Figure 4 presents the L_1 phase biases obtained for the first calibration session along with the clock offset estimate. Even if the fractional part of a float number is within a range of [0, 1], the results have been transformed into a range of [0.5, -0.5]. This approach was initially used in Gabor [1999] and the established “convention” has been kept.

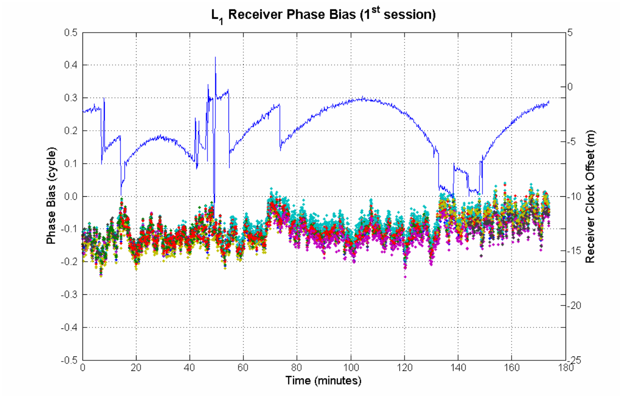

Figure 4: Receiver phase biases estimated from the first calibration session

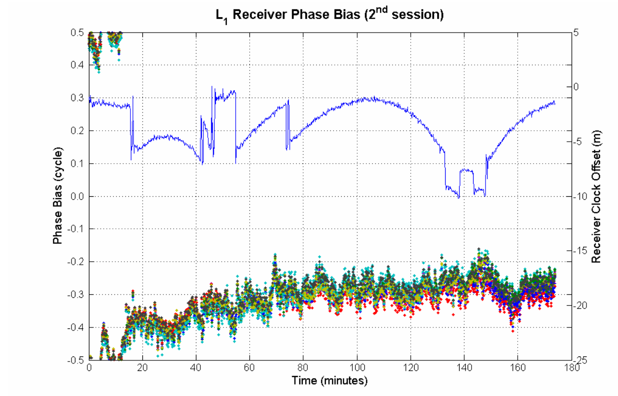

Figure 5: Receiver phase biases estimated from the second calibration session

Each series of a distinct color represents the receiver phase bias of a particular satellite. As one can see, all satellites have almost identical biases, which is logical since it has been known for a long time that differential techniques (between satellites) greatly reduce receiver hardware delay effects. A slight difference can be noticed between the series which could possibly be caused by channel dependant delays. Moreover, a drift is visible: initial values around -0.15 cycles are observed while they end at approximately -0.05 cycles after a three-hour period, which could be caused by thermal effects. Finally, the correlation between the receiver clock and the ambiguity parameters is noticeable. When a clock slew happens, even if the ambiguities preserve the same integer value, all fractional parts are subject to an identical jump and then converge back to the mean value. Further investigations are needed to understand more adequately this behavior.

Figure 5 shows the L_1 phase biases of the second calibration session made a day later. The mean value of the biases is clearly different as compared to the one from the previous session, which confirms the results obtained by Wang and Gao [2007] indicating that the biases are different when the receiver is turned off and back on. On the other hand, it is not possible to conclude with certitude that the phase (only) biases are modified, as it has previously been shown that code biases are not constant either without a stable external oscillator [Petit et al., 2001].

Figure 5 also shows a much more accentuated drift at the beginning of the session. In the first session, the receiver had been powered on for a certain period of time before the data recording, while for the second session, both events occurred almost simultaneously. Thermal effects (receiver warming up) then become a plausible explanation for this behavior.

Additional tests would be needed in order to get a better comprehension of the characteristics of the biases observed. For instance, using an external oscillator in a temperature-controlled environment could reveal valuable information. Currently, the most efficient solution is still to perform satellite-satellite single differencing (SSSD) to eliminate the receiver biases. This approach will then be used in calibrating the satellite phase biases, which is the topic of the next section.

SATELLITE PHASE-BIAS CALIBRATION

Satellite phase biases are certainly the most complicated delays to handle in ambiguity resolution for PPP. However, because of a possible long-term stability of these biases, several calibration methods have been proposed recently [Gabor, 1999; Ge et al., 2006; Leandro and Santos, 2006; Laurichesse and Mercier, 2007]. This section will first review the basics on which most of the existing approaches rely to get a perspective of the potential problems that could be encountered. Then, an alternate calibration method will be presented to overcome some of the limitations discovered.

The "Melbourne-Wübbena" approach

Most of the existing methods for satellite phase-bias calibration rely on the "Melbourne-Wübbena" signal combination [Melbourne, 1985; Wübbena, 1985] because it allows reducing considerably the error budget affecting the resulting observation. Expressed in satellite-satellite single difference (SSSD), denoted as ∇ in the following equations, this combination can be formed using the widelane carrier phase combination (*wl*):

$$\begin{aligned}\nabla\Phi_{wl} &= \frac{f_1\nabla\Phi_1 - f_2\nabla\Phi_2}{f_1 - f_2} \\ &= \nabla\bar{\rho} + \frac{f_1}{f_2} I_1 + \lambda_{wl}\left(\nabla N_{wl} + \nabla b^{\phi_{wl}}\right)\end{aligned} \tag{6}$$

and the narrowlane code combination (*nl*):

$$\begin{aligned}\nabla P_{nl} &= \frac{f_1 \nabla P_1 + f_2 \nabla P_2}{f_1 + f_2} \\ &= \nabla \bar{\rho} + \frac{f_1}{f_2} I_1 + \frac{f_1 \nabla b^{P_1} + f_2 \nabla b^{P_2}}{f_1 + f_2}\end{aligned} \tag{7}$$

Using Equations (6) and (7), the Melbourne-Wübbena combination (*mw*) can be formed as:

$$\begin{aligned}\nabla \Phi_{mw} &= \nabla \Phi_{wl} - \nabla P_{nl} \\ &= \lambda_{wl} \left(\nabla N_{wl} + \nabla b^{\phi_{wl}} \right) - \frac{f_1 \nabla b^{P_1} + f_2 \nabla b^{P_2}}{f_1 + f_2}\end{aligned} \tag{8}$$

where the newly introduced terms are

$\bar{\rho}$	the geometric range combined with all non-frequency-dependant terms (m)
f_i	the carrier frequency *i* (Hz)

Equation (8) has often been used in the past to compute the widelane ambiguity and to get an estimate of the widelane phase bias. It is important to note though that the narrowlane code biases also present in the equation will not only change the fractional part of the ambiguity estimated, but will also contribute to an integer portion of the computed ambiguity. Using Equation (8), the resulting ambiguity can then be expressed as:

$$\nabla \bar{N}_{mw} = \nabla N_{wl} + \nabla b^{\phi_{wl}} - \frac{1}{\lambda_{wl}} \nabla b^{P_{nl}} \tag{9}$$

Combining, at this stage, the estimated ambiguities with the ambiguities estimated using the ionosphere-free combination to obtain the L_1 ambiguities (as suggested in some of the aforementioned references) introduces further biases. This causes the situation to become even more complex. For the sake of simplicity, only the widelane case will be analyzed in this paper. The other cases are discussed by Banville [2007].

Derivation of an Alternate Method

Can the phase biases computed using Equation (9) be used in PPP to recover the integer nature of the ambiguities? To answer this question, one must first know the nature of the biases present in the SSSD widelane observable. According to Equation (6), it seems like the only bias present is the widelane phase bias. However, the satellite clock corrections ($\overline{dt}$), currently estimated using the ionosphere-free combination by the IGS or the GPS control segment, introduce the ionosphere-free (*if*) code biases in the observables [Collins et al., 2005]; that is to say:

$$\overline{dt} = dt + b^{P_{if}} = dt + \frac{f_1^2 P_1 - f_2^2 P_2}{f_1^2 - f_2^2} \tag{10}$$

Unlike with code observables, applying the group delay correction included in the broadcast message or estimated by the IGS will not completely remove the contribution of the biases introduced in the phase measurements. The code biases being unique to each satellite, they will merge with the ambiguity parameters and add a contribution to the values estimated.

By combining Equations (6) and (10), the SSSD widelane ambiguities estimated with PPP become:

$$\nabla \bar{N}_{PPP} = \nabla N_{wl} + \nabla b^{\phi_{wl}} - \frac{1}{\lambda_{wl}} \nabla b^{P_{if}} \tag{11}$$

It seems logical then that, in order to recover the integer nature of the ambiguities in a PPP positioning context, one would have to remove the biases included in Equation (11) or, at least, their fractional contribution. It is also obvious that the ambiguities obtained from both approaches (Equation (9) and (11)) will be different. This difference can be expressed as:

$$\begin{aligned}\nabla \bar{N}_{mw} - \nabla \bar{N}_{PPP} &= \frac{1}{\lambda_{wl}} \left(\nabla b^{P_{nl}} - \nabla b^{P_{if}} \right) \\ &= \frac{1}{\lambda_{wl}} \frac{f_1 f_2}{f_1^2 - f_2^2} \left(\nabla b^{P_2} - \nabla b^{P_1} \right)\end{aligned} \tag{12}$$

The term between the parentheses corresponds to the SSSD differential code bias (DCB) between P_1 and P_2.

According to the previous results, two calibration scenarios are conceivable to obtain satellite phase biases (that obviously contain code biases as well) that would allow recovering of the integer nature of ambiguities with respect to the hardware delays present in the observations:

1) Directly use the PPP functional model without any explicit code/phase combination. This method will however be more sensitive to observational errors such as atmospheric effects, orbital errors, etc.
2) Use the Melbourne-Wübbena combination of Equation (8) and apply the correction described by Equation (12).

Practical Comparison of the Methods

The mathematical proof of the preceding subsection can be validated using a simple empirical test. The GPS observations collected at four IGS stations (NRC1, CAGS, GODE and USNO) from January 8th to 10th 2007 have been used for this purpose (see Figure 6).

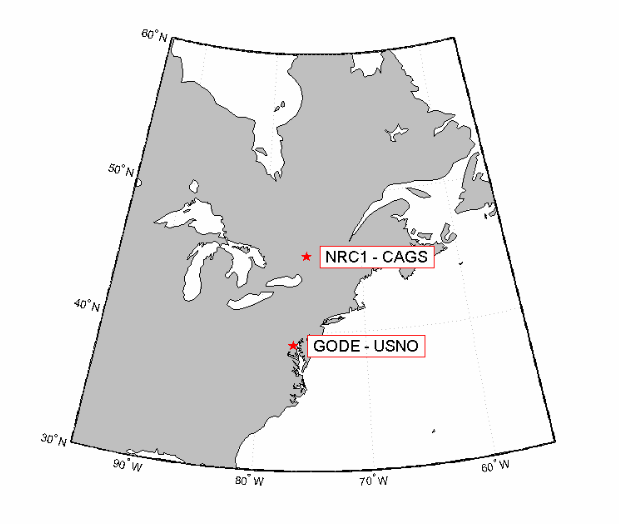

Figure 6: IGS stations used for the satellite phase-bias calibration test

First, using the PPP software developed by the first author at Laval University, the following parameters have been estimated independently at every station for each day of the test:

- constrained coordinates
- wet tropospheric zenith delay
- stochastic ionospheric delays (1/satellite/epoch using constraints from Global Ionospheric Maps (GIM) [IGS products, 2007])
- receiver clock bias
- L_1 and L_2 ambiguities (combined later on to form the widelane ambiguities)

The IGS stations were chosen in pairs forming baselines of about 20 km each. This strategy allowed exploiting the use of integer double differenced ambiguities as constraints on the undifferenced ambiguities, as well as constraining the relative atmospheric delays between the stations. However, for the test described in this paper, this approach is not explored in further detail.

Then, the same data has been reprocessed using the Melbourne-Wübbena linear combination of Equation (8). This approach allowed us to compute for each station a value of the widelane ambiguity for each satellite at each epoch. Then, an average of all ambiguity values computed for a particular station for a single satellite pass has been performed in order to reduce the noise.

In both cases, the SSSD ambiguities were formed with respect to satellite PRN 14. Then, the ambiguity values coming from both methods were differenced as suggested by Equation (12). The results are shown in Figure 7. Each symbol on the graph represents a single pass difference between the PPP-estimated widelane ambiguity and the one computed from the Melbourne-Wübbena combination for each station. One can clearly see that the differences can reach several widelane cycles (1 cycle ≈ 86 cm).

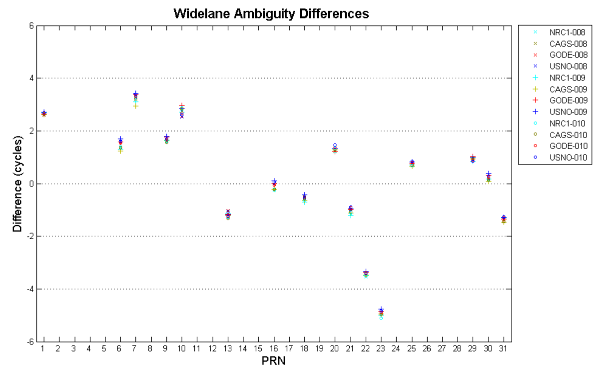

Figure 7: Difference between the PPP-estimated widelane ambiguities and the ones computed from the Melbourne-Wübbena combination

In order to confirm that the differences observed match Equation (12), the DCB values were taken from the IONEX files of January 8th 2007 [IGS products, 2007] and then scaled using the previously mentioned equation. The results are shown in Figure 8.

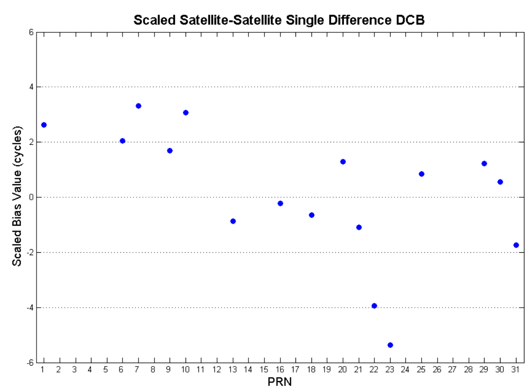

Figure 8: SSSD DCB from January 8th 2007, scaled according to Equation (12)

The two figures show good agreement, which leads us to believe that the biases affecting each estimation technique have been correctly identified in Equations 8 and 11.

CONCLUSION AND FUTURE WORK

Receiver and satellite phase biases are in a great part responsible for the problems related to ambiguity resolution in PPP. Even though several techniques can be used to deal with those biases, this paper opted for the calibration route. Thus, calibration methods for both types of biases were presented and a special attention has been paid to correctly handle the impact of code biases on the estimated values.

Receiver phase-bias calibration has been performed using a GPS signal simulator. However, the receiver code biases contaminated the receiver clock estimation which, in turn, affected the estimated ambiguities. The tests also allowed the confirmation of the results of previous studies showing that the receiver phase bias is not constant after a receiver re-initialization. Additional tests using an external oscillator in a temperature-controlled environment could allow valuable information to be obtained on the receiver biases. One should also keep in mind that the signal simulator introduces further biases which are hard to quantify.

Finally, the use of the Melbourne-Wübbena combination has been discussed for the satellite phase-bias calibration. It has been shown that an additional correction would be needed in order to use the bias computed with this combination with PPP's functional model. The alternate calibration method presented in this paper considers code biases with a special care in order to estimate coherent phase biases. Only the case of the widelane has been presented in this paper, but the rationale behind this method can be used to estimate the L_1 and L_2 phase biases as well. In the future, tests should be performed to assess the performance of this method on ambiguity resolution success rate.

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