An Optimized Least-Squares Technique for Improving Ambiguity Resolution and Computational Efficiency

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BIOGRAPHIES

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ABSTRACT

In GPS carrier phase integer ambiguity search methods, the number of ambiguity candidates to be searched and verified can be an important factor for the performance of ambiguity resolution and computational efficiency. The key question in assessing such methods is by how much and with what efficiency can the number of candidates be reduced before or at the search-verification step. The most effective procedure can be found in such techniques as a search space (or domain) transformation and an ambiguity candidate filtering (or conditioning) in multi-search levels.

An Optimal Method for Estimating GPS Ambiguities (OMEGA) that enables very high performance and computational efficiency has been developed and demonstrated. This method employs two search space reduction processes – a scaling and a screening process – that are related to the search space transformation and the ambiguity candidate filtering in multi-search levels. To obtain the highest efficiency, an optimization procedure, which decides the parameters to minimize the candidates under given conditions, is implemented in closed-form before the search-verification step.

The method is essentially based on the least-squares-approach originally proposed by Hatch but uses a modified and more efficient process. The simple test results reported in this paper have shown that computational efficiency is improved by about 90% when compared with that of the basic least-squares-approach.

INTRODUCTION

In navigation and surveying systems using GPS carrier phase data, of great concern are the performance of ambiguity resolution and computational efficiency. These parameters are often traded off in designing the system. One possible way to overcome the trade-off is to reduce the number of ambiguity candidates before or at the search-verification step. The search space transformation [*Abidin,* 1993; *Teunissen,* 1994; *Martin-Neira et al.*, 1995] and ambiguity candidate filtering in multi-search levels [*Chen and Lachapelle*, 1995; *Teunissen*, 1997] are effective techniques for that purpose.

When we use a process similar to the least-squares-approach of *Hatch* [1990] at the search-verification step, it is possible to implement optimization procedures reducing the number of candidates before implementing the step. We will show that this can be achieved using the design matrix of the linearized double-difference observables. The aim of this paper is to describe the theoretical concepts of a new on-the-fly ambiguity resolution technique – OMEGA (Optimal Method for Estimating GPS Ambiguities).

The GPS Observables

To simplify discussions we will assume that the float estimates of ambiguities and their error models are given.

For the double-difference observables recorded on short baselines, the satellite and the receiver clock biases are removed, and the residual atmospheric effects are negligible. Ignoring multipath, we have

$$\begin{aligned} \mathbf{l} &= \mathbf{Ax} + \mathbf{N} + \mathbf{e} \\ E[\mathbf{e}] &= \mathbf{0}, \quad Cov[\mathbf{e}] = \mathbf{Q} \end{aligned} \tag{1}$$

where $\mathbf{l}$ is the nx1 misclosure vector of the difference between the double-difference observations and their estimates; n is the number of the double-difference observations; $\mathbf{x}$ is the 3x1 vector of the unknown remote station position components; $\mathbf{A}$ is the design matrix for the unknown position; $\mathbf{N}$ is the nx1 vector of ambiguity parameters; $\mathbf{e}$ is the nx1 vector of the double-difference observation noise; $E[\cdot]$ and $Cov[\cdot]$ represent the mathematical expectation and the variance-covariance operators.

The Modified Least-Squares-Approach

Using the same terminology as *Hatch* [1990], we outline the modified process for the least-squares-approach in Table 1. In the computational equations, the subscripts of "p" and "s" represent the primary and the secondary group of satellites; and $round[\cdot]$ is the rounding to the nearest integer operator.

When compared with the original least-squares-approach using the double-difference observable (more details in *Erickson* [1992]), the new process gives exactly the same residuals. It is possible however, for the modified process to employ search space reduction processes when an optimal estimation problem for the secondary innovations vector is considered.

Table 1. Summary of the modified least-squares-approach.

Processing steps	Computational equations
Potential solutions	$\mathbf{x_p} = -\mathbf{A_p^{-1}N_p}, \quad \mathbf{N_p} \in Z^3$
Secondary ambiguities	$\mathbf{N_s} = round\left[\mathbf{l_s} - \mathbf{A_s A_p^{-1}(l_p - N_p)}\right]$
Innovations vector	$\mathbf{l'_p} = \mathbf{l_p}$ $\mathbf{l'_s} = \mathbf{l_s} + \mathbf{A_s A_p^{-1} N_p} - \mathbf{N_s}$
Residuals	$\mathbf{v} = (\mathbf{I} - \mathbf{AA^*})\mathbf{l'}$ $\mathbf{A^*} = (\mathbf{A^T Q^{-1} A})^{-1}\mathbf{A^T Q^{-1}}$ $\mathbf{l'} = \left[\mathbf{l_p'^T} \quad \mathbf{l_s'^T}\right]^T$

THE SEARCH SPACE REDUCTION PROCESSES

When we compute the residuals using the equations in Table 1, the only variable parameter is the secondary innovations vector. In accord with the least-squares principle, the optimal estimate for the secondary innovations vector is given as:

$$\hat{\mathbf{l}}'_s = \mathbf{A_s A_p^{-1} l_p} \tag{2}$$

We can recognize that the optimal estimate is independent of the search-verification step, since it is derived by the design matrix and by the misclosure vector for the primary group. These parameters are constant in a snapshot (i.e., single epoch) approach, such as the least-squares-approach.

The significance of the optimal estimate is that it can be used in the derivation of the scaling and screening process.

The Filter for the Secondary Innovations Vector

To derive the scaling process, we will define the filter using the optimal estimate as:

$$|w_i| \le \tau_i, \quad i = 1, 2, \cdots, n-3 \tag{3}$$

with a filter equation:

$$\begin{aligned} \mathbf{w} &= \mathbf{l'_s} - \hat{\mathbf{l}}'_s \\ &= \mathbf{l_s} - \mathbf{A_s A_p^{-1}(l_p - N_p)} - round\left[\mathbf{l_s} - \mathbf{A_s A_p^{-1}(l_p - N_p)}\right] \end{aligned} \tag{4}$$

and where τ_i is the threshold for observation i which is derived from the error model of the filter equation. In general, the ellipsoidal region in R^{n-3} centered at the estimate $\hat{\mathbf{x}}$ of a certain vector $\mathbf{x}$ can be expressed as:

$$(\mathbf{x} - \hat{\mathbf{x}})^\mathbf{T} \mathbf{Q}_{\hat{\mathbf{x}}}^{-1} (\mathbf{x} - \hat{\mathbf{x}}) \le \chi^2 \tag{5}$$

where χ^2 is a positive constant which can be determined from the probability distribution of $\mathbf{x}$; and $\mathbf{Q}_{\hat{\mathbf{x}}}$ is the variance-covariance matrix of $\hat{\mathbf{x}}$. One simple way to set the threshold is

$$\tau_i = \sigma_i \sqrt{\chi^2} \tag{6}$$

where σ_i is the square root of the i^{th} diagonal element in $\mathbf{Q}_{\hat{\mathbf{x}}}$.

Table 2. Error models for the filter thresholds.

$\hat{\mathbf{x}}$	$\mathbf{Q}_{\hat{\mathbf{x}}}$
$\mathbf{l_s}$	$\mathbf{Q_{ss}}$
$\mathbf{l_s} - \mathbf{A_s A_p^{-1} l_p}$	$\mathbf{Q_{ss}} + \mathbf{A_s A_p^{-1} Q_{pp} A_p^{-T} A_s^T} - \mathbf{A_s A_p^{-1} Q_{ps}} - \mathbf{Q_{sp} A_p^{-T} A_s^T}$
$\mathbf{A_s A_p^{-1} l_p}$	$\mathbf{A_s A_p^{-1} Q_{pp} A_p^{-T} A_s^T}$

We have tested three error models for the filter thresholds in Table 2: (a) the first represents a wrong error model; (b) the second is the case that the filter equation can be interpreted as a real-valued vector minus its integer estimate; and (c) the third error model is based on the fact that the secondary innovations vector can be predicted using the optimal estimate. Figure 1 shows the performance of the filter thresholds for each error model.

In each error model, the variance-covariance matrices with subscripts are the sub-matrices partitioned from that of the double-difference observations in equation (1).

We need in general an error model that protects all the values of the filter equation for true ambiguities, which is also efficient. The third error model clearly satisfies these criteria as shown by the examples in Figure 1.

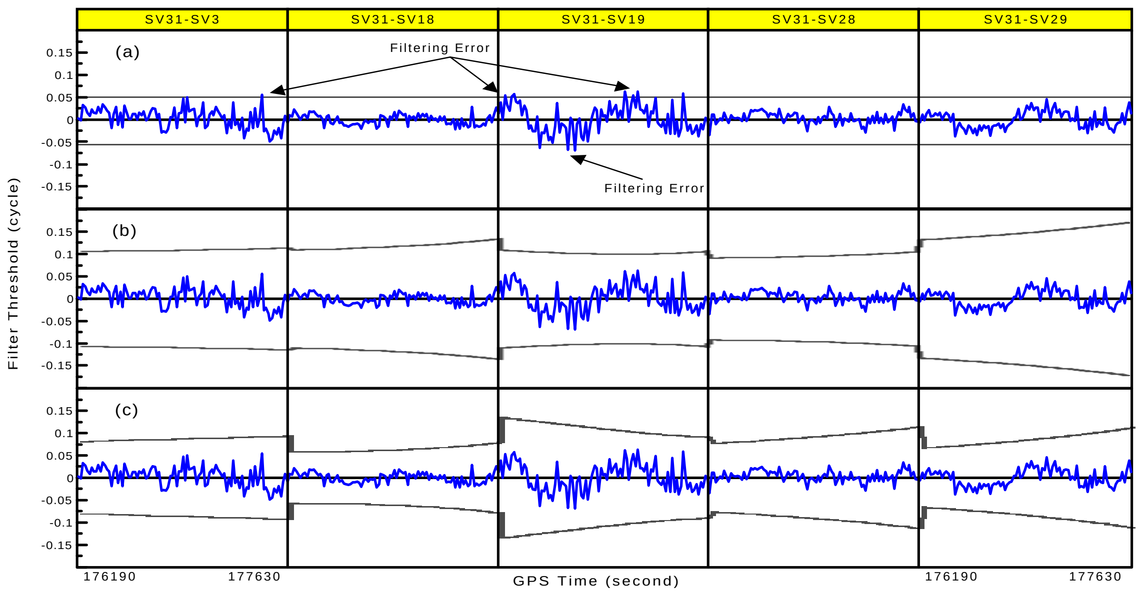

Figure 1. Performance of the filter thresholds for the filter equation.

The Scaling Process

For simplicity, we will consider equation (4) as:

$$\mathbf{w} = \mathbf{SN_p} + \nabla - round\left[\mathbf{SN_p} + \nabla\right] \tag{7}$$

where

$$\begin{aligned} \mathbf{S} &= \mathbf{A_s A_p^{-1}} \\ \nabla &= \mathbf{l_s} - \mathbf{Sl_p} \end{aligned} \tag{8}$$

Using the terms in equation (7), we can define an integer transformation and an inverse transformation as:

$$\mathbf{d_I} = round\left[f\left(\mathbf{N_p}\right)\right] = round\left[\mathbf{SN_p} + \nabla\right] \tag{9}$$

$$\mathbf{N_p} = g\left(\mathbf{d_I}\right) = round\left[\mathbf{S}^*(\mathbf{d_I} - \nabla + \tau)\right] \tag{10}$$

where $\mathbf{S}^*$ is the generalized inverse matrix of $\mathbf{S}$. The necessary and sufficient condition that the inverse transformation exists is

$$rank[\mathbf{S}] \geq \dim[\mathbf{N_p}] \tag{11}$$

where $rank[\cdot]$ and $\dim[\cdot]$ are the rank of a matrix and the dimension of a vector.

As shown in Figure 2, the function f in equation (9) represents a linear transformation from the ambiguity candidate domain onto the range space scaled by the factors in **S**. This function holds a one-to-one mapping relationship for all the candidates; therefore, there is no scaling effect. The practical scaling effect can be gained by the rounding operation. In this case, the mapping relationship is changed into many-to-one for all subspaces in the ambiguity candidate domain, where the subspace is defined as the group of ambiguity candidates which can be transformed to the same $\mathbf{d_I}$. It is possible, therefore, to estimate the bounding search range of the scaled search space as:

$$round\left[-|\mathbf{S}|\mathbf{W_p} + \nabla\right] \leq \mathbf{d_I} \leq round\left[|\mathbf{S}|\mathbf{W_p} + \nabla\right] \tag{12}$$

where $\mathbf{W}_\mathbf{p}$ represents a vector for $\mathbf{N}_\mathbf{p}$ which includes the predefined search ranges.

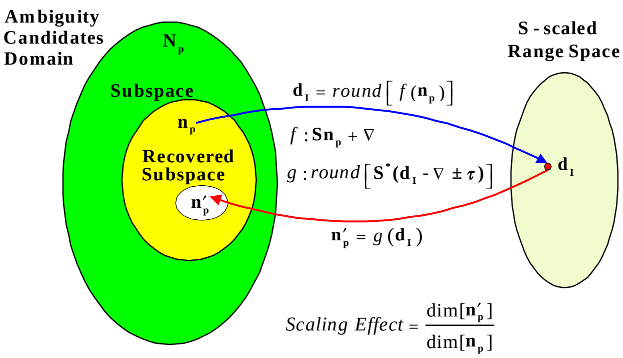

Figure 2. Diagram of the scaling process.

The inverse linear transformation g in equation (10) recovers all the candidates satisfying equation (3). In this case, the scaling effect gained from equation (9) can be affected by the factor $\mathbf{S}^*\boldsymbol{\tau}$. This factor can be considered as a magnification factor, because it usually magnifies the scaled search space. This is why we need a more efficient error model for the filter thresholds.

The Screening Process

Previous studies utilizing the inner constraints of the multi-search levels [e.g. *Chen and Lachapelle*, 1995; *Teunissen*, 1997] have shown that the ambiguity parameters of lower search levels can be conditioned on those of upper search levels. The screening process, which can be derived from the definition of the optimal estimate, has similar features.

To derive the screening process, we rewrite equation (2) using the scaling matrix as:

$$\hat{\mathbf{l}}'_s = \mathbf{S}\mathbf{l}_p = \mathbf{S}_1\mathbf{l}_{p1} + \mathbf{S}_2\mathbf{l}_{p1} \tag{13}$$

where the subscripts of “1” and “2” are the indicators of partitioned matrices and vectors. The first partitioned vector is given as:

$$\mathbf{l}_{p1} = \tilde{\mathbf{N}}_{p1} - \mathbf{N}^0_{p1} \tag{14}$$

where $\tilde{\mathbf{N}}_{p1}$ is the vector of float ambiguities and $\mathbf{N}^0_{p1}$ is the vector of the integer estimates. When the first partitioned vector is known, we can express equation (13) as:

$$\hat{\mathbf{l}}'_s = \mathbf{S}_1\mathbf{l}^F_{p1} + \mathbf{S}_2\mathbf{l}_{p2|1} \tag{15}$$

with the known vector:

$$\mathbf{l}^F_{p1} = \tilde{\mathbf{N}}_{p1} - \mathbf{N}^F_{p1} \tag{16}$$

and where $\mathbf{l}_{p2|1}$ represents the unknown vector conditioned on the known values of the first partitioned vector; and $\mathbf{N}^F_{p1}$ is the vector of the known ambiguity parameters.

It is possible therefore, to derive a relational equation for the unknown conditioned vector. Using the equality of equation (13) and (15), we have

$$\mathbf{l}_{p2|1} = \mathbf{l}_{p2} + \mathbf{S}^*_2\mathbf{S}_1\mathbf{N}_{p1} \tag{17}$$

where

$$\begin{aligned} \mathbf{S}^*_2 &= (\mathbf{S}^T_2\mathbf{S}_2)^{-1}\mathbf{S}^T_2 \\ \mathbf{N}_{p1} &= \mathbf{N}^F_{p1} - \mathbf{N}^0_{p1} \end{aligned} \tag{18}$$

We can recognize that the unknown conditioned vector can be shifted; and the shifting quantity depends on the matrix $\mathbf{S}$ and the known ambiguity parameters. The variance-covariance matrix of equation (17) is given as:

$$Cov\left[\mathbf{l}_{p2|1}\right] = Cov\left[\mathbf{l}_{p2}\right] \tag{19}$$

Figure 3 shows that this process is simply screening the predefined search space in accord with equation (17). The bounding search range of the screened search space can be estimated as:

$$\max\left[-\mathbf{W}_{p2}, \mathbf{l}_{p2|1} - \mathbf{W}_{p2}\right] \le \mathbf{N}_{P2|1} \le \min\left[\mathbf{W}_{p2}, \mathbf{l}_{p2|1} + \mathbf{W}_{p2}\right] \tag{20}$$

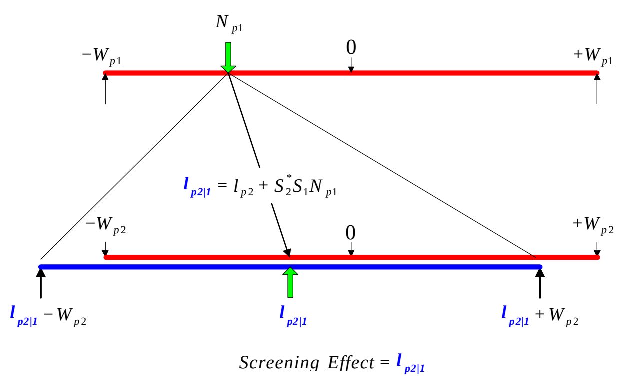

Figure 3. Diagram of the screening process.

THE OPTIMIZATION PROCEDURES

There are three parameters for reducing the search space: the scale factor **S** and the magnification factor $\mathbf{S}^*\tau$ in the scaling process, and the shifting quantity $\mathbf{S}_2^*\mathbf{S}_1\mathbf{N}_{\mathbf{p1}}$ in the screening process. As we can notice, these parameters are related to the matrix **S**. To obtain an optimal solution, we can define an objective function – the total search space volume as:

$$\Omega = \prod_{i=1}^{3} 2W_i \tag{21}$$

where W_i is the search range to be redefined at the ith search level and the number of the search levels is given as 3, which comes from the dimension of the ambiguity parameters vector. To optimize the objective function, two schemes can be employed for these parameters: the scheme of grouping the satellites into primary and secondary satellites, and the manipulating scheme of matrix **S**, including ordering, partitioning, and dimensioning.

Optimization of the Matrix S

Regarding the satellites grouping scheme, *Hatch* [1990] proposed choosing four satellites that have a reasonably good GDOP as the primary group. It is possible however, to decide the grouping in the optimization problem – finding any parameter to maximize the scaling effect or to minimize total search space volume.

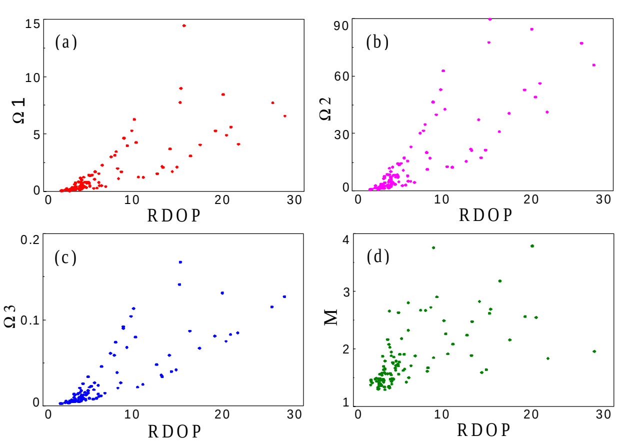

Figure 4. RDOP as the optimal parameter to minimize total search space volume: (a) objective function Ω_1 for the scaling effect, (b) modified objective function Ω_2 for the scaling effect, (c) objective function Ω_3 for the filter thresholds, and (d) magnification factor M.

For simplicity, we will assume that all search levels have a predefined search range of the same size w_p. Using the elements of the matrix **S**, we can express the scaled search range as:

$$W_i = w_p \sum_{j=1}^{3} |s_{ij}| \tag{22}$$

Substituting equation (22) into (21) and ignoring constant value $8w_p^3$ give

$$\Omega_1 = \prod_{i=1}^{3}\left(\sum_{j=1}^{3} |s_{ij}|\right) \propto \Omega \tag{23}$$

To derive a matrix expression, we can define a modified objective function as:

$$\Omega_2 = \sum_{i=1}^{3}\left(\sum_{j=1}^{3} s_{ij}^2\right) \tag{24}$$

It can be proved that any parameter minimizing Ω_2 minimizes Ω_1, but the inverse is not the case. We have therefore,

$$\Omega_2 = tr\left[\mathbf{S}\mathbf{S}^{\mathrm{T}}\right] \propto tr\left[(\mathbf{A}_{\mathbf{p}}^{\mathrm{T}}\mathbf{A}_{\mathbf{p}})^{-1}\right] \cong RDOP^2 \tag{25}$$

where $tr[\cdot]$ is the matrix trace operator, and RDOP represents the relative dilution of precision to assess the geometrical strength of the actual satellite configuration for relative positioning [*Seeber*, 1993].

In the inverse transformation of equation (10), the total search space volume can be affected by the magnification factor $\mathbf{S}^*\tau$. When the same value is assumed for the filter thresholds, we have the magnification factor as:

$$M = \left\{\tau^{n-3}\right\} \cdot \left\{\prod_{i=1}^{n-3}\left(\sum_{j=1}^{n-3} |s_{ij}^*|\right)\right\} \tag{26}$$

where we have separated the effect of the filter thresholds and the inverse matrix $\mathbf{S}^*$ using brackets for clarity. For the best RDOP, the inverse matrix $\mathbf{S}^*$ takes the opposite effect to equation (23). To see the effect of the filter thresholds, we can take the trace operation for the error model of (b) or (c) in Table 2. We have therefore,

$$\Omega_3 = tr\left[\mathbf{Q}_{\hat{x}}\right] \propto tr\left[(\mathbf{A}_{\mathbf{p}}^{\mathrm{T}}\mathbf{Q}_{\mathbf{pp}}^{-1}\mathbf{A}_{\mathbf{p}})^{-1}\right] \equiv RDOP^2 \tag{27}$$

It is evident that there is a sort of cancellation effect between the filter thresholds and the inverse matrix $\mathbf{S}^*$ in this process. We can say therefore, that the four satellites with the best RDOP give the largest scaling effect. Figure 4 shows that the smaller the RDOP value, the smaller the objective function and the magnification factor.

Ordering of the Matrix S

Ordering is the procedure of deciding the order of the search levels and the secondary observations. This scheme is used to search all possible combinations in the matrix **S**. Using an explicit matrix-vector expression, we can rewrite the filter equation of equation (7) as:

$$\begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_{n-3} \end{bmatrix} = \begin{bmatrix} s_{11} & s_{12} & s_{13} \\ s_{21} & s_{22} & s_{23} \\ & \vdots & \\ s_{n-3,1} & s_{n-3,2} & s_{n-3,3} \end{bmatrix} \begin{bmatrix} N_{p_1} \\ N_{p_2} \\ N_{p_3} \end{bmatrix} + \begin{bmatrix} \nabla_1 \\ \nabla_2 \\ \vdots \\ \nabla_{n-3} \end{bmatrix} - \begin{bmatrix} d_{I_1} \\ d_{I_2} \\ \vdots \\ d_{I_{n-3}} \end{bmatrix} \quad (28)$$

There are two swapping rules: First, if the columns of the matrix **S** are swapped, the order of the elements in the ambiguity parameter vector $\mathbf{N_p}$ should be changed in accord with the swapped order; in this case, there is no change of the column vectors – $\mathbf{w}$, ∇, and $\mathbf{d_I}$. Second, if the rows of the matrix **S** are swapped, the order of the elements in the column vectors should be changed according to the swapped order; there is no change in the ambiguity parameter vector.

Partitioning of the Matrix S

The screening process of equation (17) is related to the partitioned sub-matrices $\mathbf{S_1}$ and $\mathbf{S_2}$. The partitioning scheme is used therefore, to implement the screening process. The rule for this scheme is that the matrix **S** should be partitioned in the column direction. We can consider three patterns in Table 3.

Table 3. Patterns of the partitioning scheme.

1	2	3
$\mathbf{S_1} = 0$, $\mathbf{S_2} = \mathbf{S}$	$\mathbf{S_1} = \begin{bmatrix} s_{11} \\ \vdots \\ s_{n-3,1} \end{bmatrix}$, $\mathbf{S_2} = \begin{bmatrix} s_{12} & s_{13} \\ \vdots & \vdots \\ s_{n-3,2} & s_{n-3,3} \end{bmatrix}$	$\mathbf{S_1} = \begin{bmatrix} s_{11} & s_{12} \\ \vdots & \vdots \\ s_{n-3,1} & s_{n-3,2} \end{bmatrix}$, $\mathbf{S_2} = \begin{bmatrix} s_{13} \\ \vdots \\ s_{n-3,3} \end{bmatrix}$

The first pattern is not for practical implementation, but for general expression. The second represents the case that the ambiguity parameters in the first search level are known. When the ambiguity parameters in the first and second search levels are known, the third pattern is used.

Dimensioning of the Matrix S

The dimensioning scheme is considered for the scaling process. In the integer transformation defined as equation (9), the number of the transformed search levels (i.e., the dimension of $\mathbf{d_I}$) is given as n-3. Under the condition of equation (11), the minimum dimension can be obtained as 3 if and only if we use a square-matrix **S** with rank 3, where the number 3 comes from the dimension of the ambiguity parameters vector.

Considering the situation when the scaling process is implemented in the multi-search levels, the dimension of the unknown ambiguity parameters vector will be changed according to the search levels; therefore, we need to adjust the square-matrix **S** in accord with the dimension. This means that a square sub-matrix $\mathbf{S_{sub}}$ is chosen from the matrix **S**. In general, the dimensioning scheme has priority over the partitioning scheme. We can consider three patterns in Table 4.

Table 4. Patterns of the dimensioning scheme.

1	2	3
$S_{sub} = \begin{bmatrix} s_{11} & s_{12} & s_{13} \\ s_{21} & s_{22} & s_{23} \\ s_{31} & s_{32} & s_{33} \end{bmatrix}$	$S_{sub} = \begin{bmatrix} s_{12} & s_{13} \\ s_{22} & s_{23} \end{bmatrix}$	$S_{sub} = s_{13}$

The following describes practical implementation methods that can be considered in the optimization procedures:

Method 1: Choose the first dimensioning pattern. In this case, only the first pattern of the partitioning scheme is available; therefore only the scaling process can be used. The advantage of this method is that the scaled search space can be defined independently before the search loops. The scaled search space is however, not larger than the predefined search space. This method is not practical but conceptual.

Method 2. Choose the second dimensioning pattern. In this case, the first and second patterns of the partitioning scheme are available and the first search level is set using the predefined search range. Using all the ambiguity candidates in the first search level, the screening process is implemented to the second and third search levels. And then, the scaling process is implemented for the screened search ranges. Although this method utilizes both search space reduction processes, the reduction effect is not large.

Method 3. Choose the third dimensioning pattern. Then all the patterns of the partitioning scheme are available. In this case, the first search level is set using the predefined search range. Using all the ambiguity candidates in the first search level, the screening process is implemented at the second search level. And then the third search level is screened using all the screened ambiguity candidates in the second search level. Finally, the scaling process is applied to the third screened search level. This method has shown the best results in our investigations. Although we must implement a few computational steps at the second and third search levels, it does not deteriorate the efficiency of this method; in fact, the reduction effect is much more predominant.

Optimization Procedures Implementation Fundamentals

As mentioned earlier, the purpose of the optimization procedures is to find the parameter that minimizes the number of ambiguity candidates before the search-verification step where the scaling and screening processes are implemented. This parameter is given as a reordered matrix **S**. Figure 5 shows these procedures.

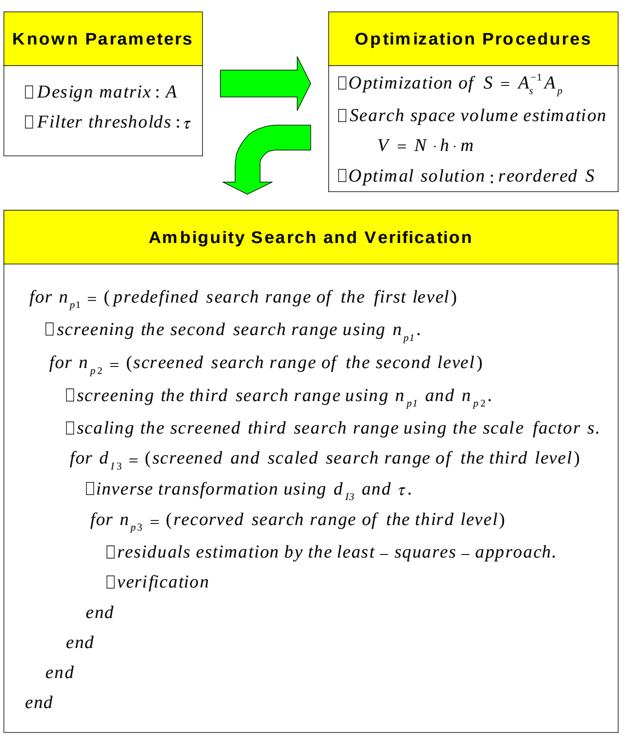

Figure 5. Ambiguity search-verification step augmenting the optimization procedures.

We will consider the situation for example, that nine satellites are observable. Then we have eight double-difference observations: three for the primary group and five for the secondary group; therefore the dimension of the matrix **S** is 5x3, where the number of the unknown parameters is 3. In this case, we have 30 combinations by: 6 column-order sets, and 5 redundancies of the observations. To find optimal parameters we have to estimate the total search space volume for each combination. Based on our investigations, we will implement method 3 in the optimization procedures.

The first step of the estimation is finding all ambiguity candidates that belong to the search plane generated by the first and second search levels that pass through the third search level. This can be done easily on account of the linearity of equation (17).

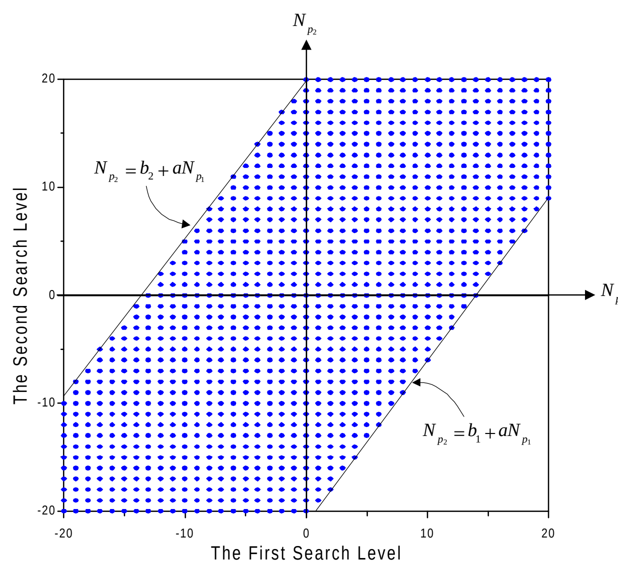

Figure 6. Screened ambiguity candidates on the search plane generated by the first and second search levels.

It is clear in Figure 6 that the number of ambiguity candidates *N*, can be estimated as the dotted area when the slope *a*, and two y-axis intersections b_1 and b_2 are given. From equations (17) to (20), we have:

$$
\begin{aligned}
a &= \left[\mathbf{S}_2^*\mathbf{S}_1\right]_1 \\
b_1 &= \max\left[-W_{p2}, l_{p2} - W_{p2}\right] \\
b_2 &= \min\left[+W_{p2}, l_{p2} + W_{p2}\right]
\end{aligned}
\qquad (29)
$$

where the slope *a* is the first element of the vector $\mathbf{S}_2^*\mathbf{S}_1$ given by the second pattern of Table 3.

As the next step, we have to estimate the screened and scaled search range at the third search level using all the ambiguity candidates *N*. It is impractical however, to estimate the search range using all the ambiguity candidates. Instead, we have used representative ambiguity candidates, which are the ambiguity candidates on the line: $N_{p_2} = 0$. For each candidate, the screened search range is given by:

$$\varpi_{31} \leq N_{p3|12} \leq \varpi_{32} \qquad (30)$$

where

$$\begin{aligned} \varpi_{31} &= \max\left[-W_{p3}, l_{p3|12} - W_{p3}\right] \\ \varpi_{32} &= \min\left[+W_{p3}, l_{p3|12} + W_{p3}\right] \\ l_{p3|12} &= l_{p3} + \mathbf{S}_2^* \mathbf{S}_1 \begin{bmatrix} N_{p1} \\ 0 \end{bmatrix} \end{aligned} \qquad (31)$$

In equation (31), the vector $\mathbf{S}_2^* \mathbf{S}_1$ follows the third pattern of Table 3. Applying the scaling process for this screened search range gives:

$$round\left[|\, s_{13} \,|\, \varpi_{31}\right] \leq d_{I3} \leq round\left[|\, s_{13} \,|\, \varpi_{32}\right] \qquad (32)$$

It is possible therefore, to estimate the mean value *h* of the screened and scaled search ranges for all the representative ambiguity candidates. The final step is estimating the magnification factor as:

$$m = 2\,|\, s_{13}^* \,|\, \tau_3 = 2\left(\frac{\tau_3}{s_{13}}\right) \qquad (33)$$

The estimate of total search space volume is therefore, given as:

$$V = N \cdot h \cdot m \qquad (34)$$

For all combinations, we can estimate the total search space volumes. The optimal solution is given therefore, as the reordered matrix S that gives a minimum volume.

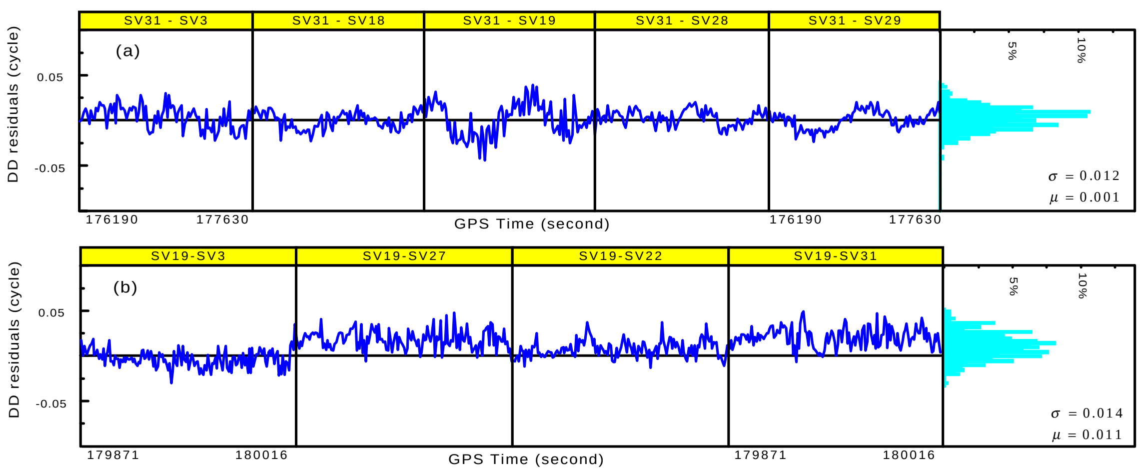

Figure 7. Double-difference residuals: (a) in static mode, and (b) in kinematic mode.

RESULTS

To illustrate the success of the method, we present two sets of sample results. The first data set contains both L1 and L2 observations recorded at a fifteen second sampling interval in static mode and the second data set contains the same observations at a one second sampling interval recorded in kinematic mode. In both modes, the data were obtained on very short baselines to investigate the performance and computational efficiency of the method without considering a correct observable noise model. The matrix **S** that is independent of the observable noise is in fact, the most significant parameter of the method. Because the baselines were so short, only L1 data were used in the analysis.

Ambiguity Resolution Performance

To test the performance of ambiguity resolution, we used a snapshot approach – estimating integer ambiguities epoch by epoch. Of the data recorded in static mode, 96 samples (i.e. observations over 1,440 seconds) with nine satellites were processed. For the data recorded in kinematic mode, 146 samples with eight satellites were processed. The success rate in identifying the correct ambiguity values of these examples was 100%. This performance is mainly due to the low noise level and high redundancy of the double-difference observations. Figure 7 shows double-difference residuals after the least-squares adjustment in both modes. Almost every plot has a distinct trace indicating the low noise level of the double-difference observable.

We have not quantified for now, the contribution of the method in the performance (i.e., success rate) of ambiguity resolution. It is evident however, that the method does help in the decision process of correctly identifying the ambiguity candidate through search space reduction. This will be investigated further in on-going research.

Computational Efficiency

Comparing with the standard least-squares-approach, Figure 8 shows the general improvement gained by the search space reduction processes. Figure 8(a) represents the search space and computational time reductions in static mode. In this example, the predefined search range is approximately 2 meters; therefore the number of ambiguity candidates is about 8,000. The differences between the search space and computational time reduction result from the additional computations implemented into the search-verification steps. Figure 8(b) shows the search range reduction at each search level. There is no reduction at the first search level. The reduction at the second search level is due to the screening process. It is up to 30% in this example. On the other hand, the third search level has more than an 80% reduction in the number of candidates because of both the screening and the scaling processes. Our investigations have shown that the scaling process gives much more reduction than the screening process.

Figure 9 and 10 show the improvement in accord with search range size and confidence level for the filter thresholds. As the search range size is increased, the improvement tends to converge to around 90%. Considering the results for the confidence levels, Figure 9 and 10 show that the more efficient the error model for the filter thresholds, the more the improvement of the search space reduction.

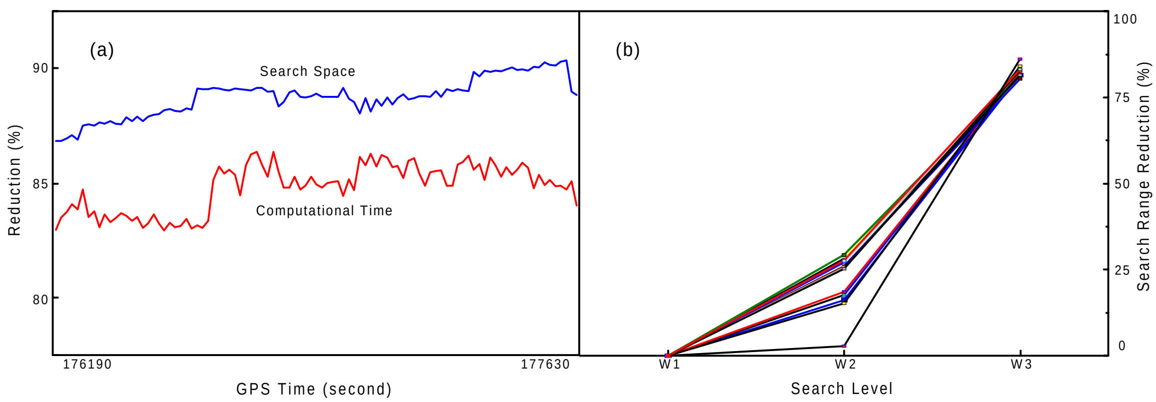

Figure 8. Computational efficiency compared with the least-squares-approach: (a) search space and computational time reduction, and (b) search range reduction at each search level.

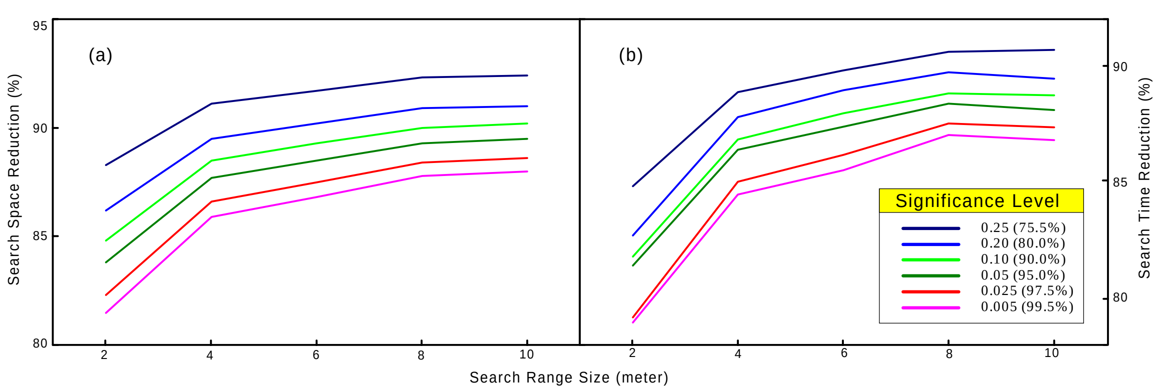

Figure 9. Computational efficiency in processing the static data according to search range size and assigned confidence level for the filter thresholds: (a) search space and (b) computational time reduction.

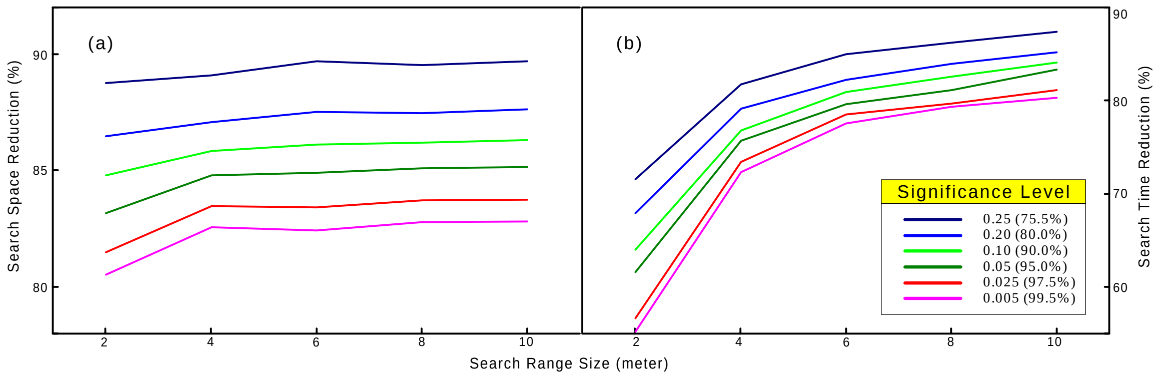

Figure 10. Computational efficiency in processing the kinematic data according to search range size and assigned confidence level for the filter thresholds: (a) search space and (b) computational time reduction.

CONCLUSIONS

We have attempted to describe in this paper the main features of a new OTF ambiguity resolution method: First, two search space reduction processes – the scaling and the screening process – can be derived by the optimal estimate for the secondary innovations vector, where the optimal estimate is determined using the constant parameters (i.e., the initial misclosure vector and the design matrix) in the snapshot approach. Second, when considering the situation that both processes are implemented in the search-verification step, the optimal parameter that minimizes total search space volume is the best RDOP. Third, total search space volume is changed according to search levels and the order of the secondary observations; therefore, it is possible to find an optimal solution in the optimization procedures, and it is given as a reordered matrix S.

Problems were encountered with the general reliability of the optimization procedures, since the procedures estimate total search space volume without counting all ambiguity candidates. The key question in this respect is how accurately the volume can be estimated. The most difficult part is estimating the search range at the final search level. By estimating the mean search range at the third search level using the representative candidates, we are assuming that the difference from the actual search range at this level is negligible.

The work reported in this paper has been only a preliminary study and further investigations are required to study error models for the filter thresholds and the contribution of the method to the overall ambiguity resolution performance. New investigations could include more efficient computational algorithms in the search-verification step.

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