

# Positioning Enhancement Based on a New Weighting Scheme to Solve an Ill-Conditioned Case

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## BIOGRAPHY

Yong-Won Ahn holds an M.Sc from the Department of Geomatics Engineering at the University of Calgary obtained in 2005. He is currently a Ph.D candidate and the part-time lecturer in the Department of Geodesy and Geomatics Engineering at the University of New Brunswick (UNB). He has been involved in GNSS research from Korea Astronomy and Space Science Institute (KASI) since 1996 in several different areas. His current research focus is the mitigation of the tropospheric anomaly on GNSS. He is also interested in developing enhanced RTK positioning software and in evaluating position solutions by simulation in different atmospheric and space-borne environments.

## ABSTRACT

The unmodelled troposphere is a limiting but crucial factor in achieving high-precision positioning solutions for dual-frequency GPS observables. Typically, the effect is minimal in a relative positioning scenario when the baseline length is short enough, e.g. less than 10 km. However, when there is strong tropospheric anomaly, the effect will not be nominal and the GNSS solution will have a much reduced precision.

As the troposphere and resultant height component are both zenith dependent, the problem exists of how to correctly de-correlate those two parameters in order to achieve better coordinate solutions especially in vertical component. There are many different approaches to solve the typical ill-conditioned case such as an orthogonal transformation. One of the innovative ways to de-correlate those parameters, proposed in this research, is to use different weighting parameters either on the troposphere or on the height component. This methodology can be realized as GPS signals can be separated in two different components: frequency-dependent and frequency-independent realm. The troposphere is of the frequency-independent domain in GPS signals, and this characteristic is to be further used as a weighting scheme through the residual analysis. The only assumption when using this method is that the both L1 and L2 observables are made

available from a dual frequency receiver. To examine the feasibility of this method, anomalous data collected in Southern Texas, USA, on August 21, 2005 over a baseline length of around 7.8 km is reprocessed. Consequently, the positioning solution from the new proposed parameter is tested, evaluated, and compared with that from the conventional estimation method. In this paper, we have examined the feasibility of our proposed method whether it also is resistant to the anomalous tropospheric case or not. By using this methodology for the tropospheric delay mitigation, positioning improvement can be achieved in 20% in the vertical component. In addition, degradation of the vertical component due to the tropospheric anomaly is significantly reduced.

## INTRODUCTION

Modernized global navigation satellite system (GNSS) signals exist at many different frequencies. One of the benefits of having more frequencies is to better model the frequency-dependent atmospheric components, for example, the ionosphere. Having with more signals and multiple choices of different frequency combination, improved ambiguity resolutions is also possible. This can make the positioning solution stronger due to having a higher resolution of the ambiguity solved. One of the most problematic error components in GNSS positioning, however, happens in the non-dispersive medium which does not depend on the current GNSS frequency ranges [Oguchi, 1983]. For this reason, even if multiple frequencies are available, they are not helpful in mitigating the tropospheric delay. Removing all frequency dependent errors (except the non-dispersive medium, such as the troposphere), can eventually improve the tropospheric modeling. This can be beneficial, but it may take significant amount of time to evaluate an entire network with recorded atmospheric profiles. In the troposphere, almost 90% of the total delay occurs in the hydrostatic component, which varies slowly with time. This hydrostatic delay can be easily modeled, with assuming hydrostatic equilibrium, to an accuracy at the millimetre level [Mendes and Langley, 1995]. Unlike the hydrostatic part, the non-hydrostatic (or wet) part has strong spatial and temporal variations. This

makes the problem very complicated. Typically, the effects of wet delay to the range direction can reach 10-40 cm depending on the elevation cutoff angle. If the functional models do not fully account for the wet delay, the resultant residual errors in modeling can cause significant errors in high-precision GNSS positioning solutions. Consequently, the misclosure vector may be too large.

To reduce or minimize the errors arising from poor modeling of the wet troposphere, one possibility is to model the tropospheric refraction using a purely independent data set, without GNSS observations, which is not always available at a receiver. The other approach is to estimate the tropospheric parameters directly using the available GNSS data by least-squares adjustment for a 1~2 hour time window, or use a Kalman filter for real-time applications. Within the Kalman filter, it may be hard to model the dynamic behavior of the troposphere and the solution is to be unexpectedly degraded once it is wrongly estimated. Due to their spatial and temporal correlation characteristics, these errors can be substantially minimized under short-baseline situations by differential techniques. Even for short baselines, however, the resultant solution can be hardly degraded once there is a strong anomaly effect due to the troposphere. The problem can be more difficult as the troposphere parameters are highly correlated with the height component which may end up with the ill-conditioned case in the normal equations. In order to attain a better positioning solution for a rover (in terms of reliability), either having a better model for the troposphere that is resistant to anomalous cases, or having a different elimination technique for the troposphere, is preferred. Current mitigation techniques, such as the theoretical or empirical model for the troposphere, are only considered in the normal troposphere cases. Some studies have focused on independent observables (e.g., water vapor from a water vapor radiometer) to retrieve the absolute atmospheric parameters for other stations. In addition, a recent achievement includes a ray-tracer based on a numerical weather prediction model (e.g., Rapid Update Cycle 13 km (RUC13) by National Oceanic and Atmospheric Administration (NOAA) in USA, and Global Environmental Multiscale (GEM) NWP model from the Canadian Meteorological Centre of Environment Canada) [Cove *et al.*, 2004; Ahn *et al.*, 2005; Cove, 2005; Nievinski *et al.*, 2005]. Currently, the grid spacing currently adopted is too large to consider locally anomalous atmospheric conditions.

In order to estimate those parameters separately in the parameter estimation process, a new recursive weighting scheme is introduced in this paper to apply proper weighting to de-correlate the troposphere and the height component to achieve high precision positioning solution in the parameter estimation process.

### Atmospheric Anomalies

From time to time, serious atmospheric anomaly effects which seriously affect GNSS positioning have been observed in many different networks around the world. These include severe sand storms, dust storms, volcanic eruptions, ionospheric scintillation effects, and localized or regional tropospheric anomaly effects. Occasionally, these phenomena are observed when a strong tropospheric anomaly exists within a network. In these cases, the position solution can be seriously degraded in both a single baseline as well as a multi-baseline network. One example of a localized anomaly was observed at Stennis Continuously Operating Reference Stations (CORS) in Texas, USA in 2005. The baseline length is about 2.1 km which is short enough to eliminate the correlated errors in the atmosphere. During the period when there was a localized troposphere anomaly, the residuals reached over a half cycle, thus causing a failure in resolving the ambiguities successfully (Lawrence *et al.*, 2006). Similar weather could be observed near San Marcos CORS stations, CSM1 and TXSM in Texas on 11<sup>th</sup> October 2005 over a 2.7 km baseline length. During the passage of the localized storm, the GPS RTK performance was highly degraded mainly due to the wrongly fixed ambiguities, resulting in corrupting the positioning performance (Ahn *et al.*, 2008). One of the large anomalies was also recorded in Southern Texas on August 21, 2005. The baseline length was of around 7.8 km. Figure 1 represents the positioning performance of the UNB-RTK software platform using the L1 frequency with fixed ambiguities. As clearly seen from the figure, the anomaly peaked at around 15:00 (Local Time). Due to the quality control of the software, almost 600 epochs during the anomaly were rejected to improve the reliable coordinate estimation.

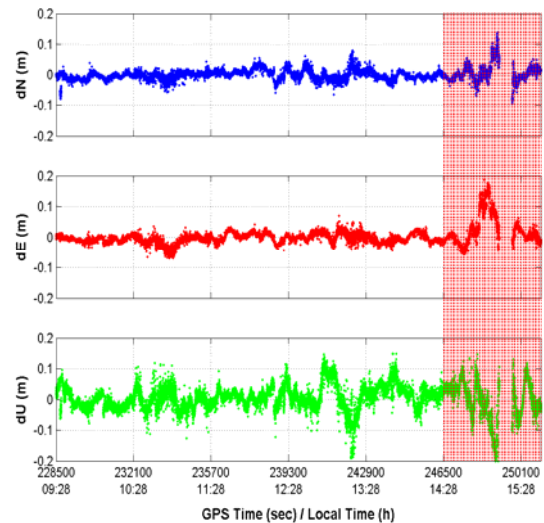


Figure 1. Kinematic Positioning Solution using UNB-RTK in a real-time scenario

We also processed the data with scientific software for further analysis. Figure 2 represents the corresponding kinematic positioning solutions obtained by Bernese GPS software version 5.0 in post processed kinematic (PPK) mode. The Bernese software pre-examines all epochs to determine the cycle slips and ambiguities, and also pre-determine stochastic parameters, such as the ionosphere. These pre-determined stochastic parameters and ambiguities can be further used to determine the “cleaned” results. Even if the whole data span was used in Bernese, we can still see that the solution is getting worse after 13:00 (local time), especially in the vertical component reaching its worst around 15:00 local time. During the processing RX1B is used as a reference, and RX2B is selected as a rover (the same as UNB-RTK). In this case in Bernese, we introduced the dry Niell mapping function with the dry Saastamoinen model [Niell, 1996; Saastamoinen, 1972], and estimated 15 minute residual tropospheric delay parameters. In order to try to obtain a better solution, we tested a few other processing strategies in Bernese. These strategies included the ionosphere-free linear combination to eliminate the first order ionosphere effect even if the noise level is almost three times higher than that of L1. This processing strategy is not usually favorable on a short baseline processing due to the noise level. We also processed the data using the L1 frequency only, without any residual tropospheric estimation and L2 or wide-lane combination etc. However, the best approach described above still has the residuals reaching up to 100 part per million (PPM) in a slant direction (line of sight) during the anomaly period.

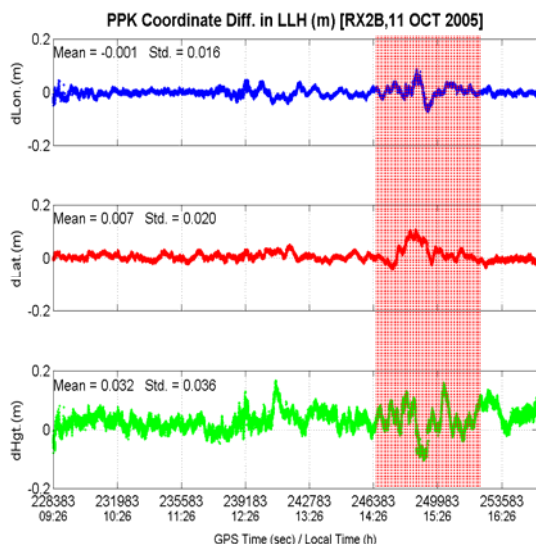


Figure 2. Kinematic positioning solutions by Bernese software using pre-processed screened results

As an example, Figure 3 represents the L1 and L2 double differenced residuals processed by Bernese software. Based on residual analysis, we can identify whether the residual of the satellite pair is caused by the ionosphere or the

troposphere. This residual identification can be further used for the weighting scheme, proposed in this research, and described in a later section.

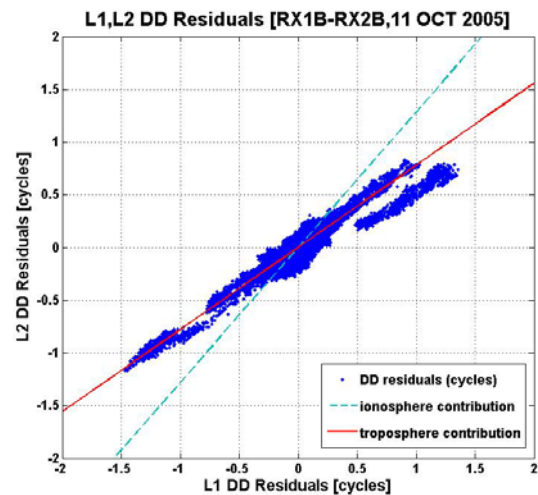


Figure 3. L1 and L2 double difference residuals comparison with Bernese PPK scenario.

The residual zenith delay of the troposphere, and the height component of the positioning solutions, is highly dependent on the zenith angle. Due to their correlation, most of the position estimation errors induced by the troposphere are amplified mainly in the vertical component. Depending on weather conditions, a stochastic modeling approach or parametric estimation method has been implemented to mitigate the tropospheric error. In order to solve the issue stated above, our previous research investigated the combined approach which can eliminate inter-correlation among zenith angle-dependent parameters and thus, improve horizontal positioning solutions (Ahn *et al.* 2008). This could result in the successful de-correlation of both the vertical component and the troposphere delay parameter. In the paper, we evaluated many different coefficients and choose when the norm of the coordinate solutions is at a minimum value as there is no strict numerical way to determine the weighting coefficient. While the arbitrary choice of a weighting coefficient is somewhat unrealistic, the approach was successful in evaluating our initial methodology. This could reduce the tropospheric residuals and resulted in a dramatic improvement in the solution domain. From a practical point of view, however, another method for determining the weighting coefficient has to be used.

## METHODOLOGY

As stated earlier, even for a short baseline, imbalanced atmospheric errors are shown to have a severe impact on rover positioning solutions, resulting in a worsening of the quality of the positioning solutions [Ahn *et al.*, 2006;

Lawrence *et al.*, 2006; Zhang and Bartone, 2006; Huang and van Graas, 2006; Kim and Langley, 2007, Ahn *et al.*, 2008]. Under extremely inhomogeneous conditions in the lower troposphere, a physical interpretation may be difficult, if not impossible to evaluate, resulting in certain misassumptions about the parametric model. Therefore, not only was a residual analysis of the tropospheric delay carried out, but also a new approach to combine the zenith dependent parameters into one common parameter is studied – this is to avoid incorrect tropospheric modeling.

In order to introduce our new weighting approach, the general mathematical background is first discussed. Under short baselines, the residual effects of the ionosphere and troposphere are typically insignificant. As we are dealing with a strong anomaly effect in the lower troposphere, the residual tropospheric term (without the assumption of atmospheric azimuthal asymmetry and use of gradient estimation) is included.

### Mathematical Background of the Combined Approach

The reduced phase equation can be abbreviated in vector-state form as follows:

$$\mathbf{L} = \mathbf{H}\mathbf{x} + \mathbf{m}\Delta\tau + \mathbf{e}, \quad \mathbf{e} \sim N(\mathbf{0}, \mathbf{Q}_L) \quad (2)$$

where

$$\begin{aligned} \mathbf{L} &= \lambda_{L1} \Phi_{AB}^{jk}(t) - \lambda_{L1} N_{AB}^{jk} - \rho_{AB_0}^{jk} - \mathbf{T}_{AB}^{jk} \\ \mathbf{H} &= [\mathbf{h}_x \quad \mathbf{h}_y \quad \mathbf{h}_z] \\ \mathbf{x} &= [\Delta X_B \quad \Delta Y_B \quad \Delta Z_B]^T \end{aligned} \quad (3)$$

where,

- $\mathbf{H}$  : Jacobian matrix
- $\lambda_{L1}$  : L1 wavelength (metres)
- $\varphi_{AB}^{jk}$  : double difference (DD) phase observables (cycles):  
superscripts j and k stand for the satellites, and  
subscripts A and B for the receivers
- $N_{AB}^{jk}$  : DD integer carrier phase ambiguities (cycles)
- $\rho_{AB}^{jk}$  : DD geometric range (metres)
- $T_{AB}^{jk}$  : DD hydrostatic (or dry) delay (meters)
- $\mathbf{m}$  :  $m_{AB}^{jk}$  double-differenced non-hydrostatic (wet) mapping coefficient (unitless).
- $\Delta\tau$  : relative wet zenith delay (metres),  $\Delta\tau_{AB}$
- $\mathbf{e}$  : residual errors (e.g., receiver system noise, multipath, etc.) which assumes it is a normally distributed random vector with expected value of  $\mathbf{0}$  and variance-covariance  $\mathbf{Q}_L$ .

In order to analyze the common zenith dependent parameters (that is, the vertical component of the receiver's position and the wet zenith delay), the local geodetic coordinate system is introduced. The axes  $n$  and  $e$  span the local geodetic horizon which is perpendicular to the ellipsoidal normal through the surface of point P.  $n$  and  $e$  point north and east, and  $u$  coincides with the ellipsoidal normal with the positive direction upwards from the ellipsoid.

The relationship between the local geodetic coordinate system and the geocentric coordinate system is as follows [Leick, 1995]:

$$\begin{aligned} \mathbf{x} &= \mathbf{R}^{-1} \mathbf{n} \\ \mathbf{R} &= \begin{bmatrix} -\sin \varphi \cos \lambda & -\sin \varphi \sin \lambda & \cos \varphi \\ -\sin \lambda & \cos \lambda & 0 \\ \cos \varphi \cos \lambda & \cos \varphi \sin \lambda & \sin \varphi \end{bmatrix} \\ \mathbf{n} &= [\Delta n \quad \Delta e \quad \Delta u]^T \end{aligned} \quad (4)$$

where  $\mathbf{R}$  is the rotation matrix and  $\mathbf{n}$  is a vector of the position component in the local geodetic system. Given the latitude and longitude of the receiver, the geocentric coordinate system can be easily transformed to the local geodetic system based on equation (4). Equation (2) can be now rewritten in a sub-matrix form as follows:

$$\begin{aligned} \mathbf{L} &= \mathbf{G}\mathbf{n} + \mathbf{m}\Delta\tau + \mathbf{e} \\ &= [\mathbf{g}_n \quad \mathbf{g}_e] \begin{bmatrix} \Delta n \\ \Delta e \end{bmatrix} + [\mathbf{g}_u \quad \mathbf{m}] \begin{bmatrix} \Delta u \\ \Delta \tau \end{bmatrix} + \mathbf{e} \\ \mathbf{G} &= \mathbf{H}\mathbf{R}^{-1} \end{aligned} \quad (5)$$

Equation (5) gives a straightforward interpretation of the vertical increment  $\Delta u$  and the wet zenith delay  $\Delta \tau$ . The vertical component of the design matrix  $\mathbf{g}_u$  and the Niell's wet mapping function coefficient  $\mathbf{m}$  [Niell, 1996] for each satellite is strongly correlated in high elevation angle and becomes weaker at lower elevation angles [Ahn *et al.*, 2008].

The challenge we try to overcome in this paper is to uncorrelated the two parameters (the vertical increment and the wet zenith delay). Even if they have a functional relationship with each other, the two parameters cannot be easily combined into one single parameter and it is probably even impossible to have a linearized relationship between them. Therefore, we follow, in this study, a numerical approach to solve the correlation problem. By introducing the new parameters,  $\alpha$  and  $\zeta$ , the two parameters can be combined as follows:

$$\mathbf{g}_u \Delta u + \mathbf{m} \Delta \tau = [\alpha \mathbf{g}_u + (1 - \alpha) \mathbf{m}] \zeta \quad (6)$$

where

$$\alpha = \frac{\Delta u}{(\Delta u + \Delta \tau)}, \quad \zeta = (\Delta u + \Delta \tau) \quad (7)$$

The  $\alpha$  represents the ratio between the vertical component increment and  $\zeta$ . From our background research, once  $\alpha$ , weighting parameter, is properly selected, the positioning is dramatically enhanced. A different value of  $\alpha$  gives a different solution as the weight different for the vertical component of the design matrix  $\mathbf{g}_u$  and Niell's wet mapping function coefficient  $\mathbf{m}$ . One difficult issue to be solved, however, is how to determine the weighing parameter,  $\alpha$ , in a practical manner. To make the problem simpler, the height estimate  $\Delta u$  is assumed to be known in this scenario. Using this scheme, the level of the contribution from the troposphere  $\Delta \tau$  in the weighting parameter can be identified in the solution domain.

### DD residual analysis for separating non-dispersive into dispersive medium and determination of their magnitudes

Once the integer or float ambiguities are determined, the residuals for L1 or L2 can be generated by the following equations:

$$R_{BA_k}^{mn} = \frac{T^m - T^n}{\lambda_{BA_k}} - (I^m - I^n)\lambda_{BA_k} + \frac{M_{BA_k}^{mn} + W_{BA_k}^{mn}}{\lambda_{BA_k}} \quad (8)$$

where,

$$R_{BA_k}^{mn} = \Phi_{BA_k}^m - \Phi_{BA_k}^n$$

$$\Phi_{BA_k}^i = \phi_{BA_k}^i - \frac{\Delta r}{\lambda_{BA_k}} - N_{BA_k}^i$$

$$M_{BA_k}^{mn} = \frac{M_{BA_k}^m - M_{BA_k}^n}{\lambda_{BA_k}}$$

$$W_{BA_k}^{mn} = \frac{W_{BA_k}^m - W_{BA_k}^n}{\lambda_{BA_k}}$$

k : a frequency (L1 or L2)

$N_{BA_k}^i$  : the double-difference cycle ambiguity for satellite i and frequency k relative to the reference satellite r for baseline A and B.

$R_{BA_k}^{mn}$  : the double-difference phase residual between satellites m and n.

$\phi_{BA_k}^i$  : the single-difference carrier-phase measurement for satellite i on frequency k (in units of cycles).

$\lambda_{BA_k}$  : the wavelength of frequency k.

$\Delta r$  : the differential distance to the satellite.

$T^i$  : the differential troposphere delay for satellite i.

$I_{\lambda_{BA_k}}^i$  : the differential ionosphere group delay at frequency k.

$M_{BA_k}^i$  : the differential carrier multipath error for satellite i and frequency k.

$W_{BA_k}^i$  : the differential receiver noise error for satellite i and frequency k.

Equation (8) shows that there is a fixed ratio between the contribution of the troposphere to the  $R_{BA_2}^{mn}$  and  $R_{BA_1}^{mn}$  residuals. Also, there is a fixed ratio between the contributions of the ionosphere to those residuals. When we choose  $R_{BA_1}^{mn}$  as the y-axis and  $R_{BA_2}^{mn}$  as the x-axis, then the troposphere contribution is given by the slope of  $\lambda_1 / \lambda_2$ . Also, in a similar way, the ionosphere can be given by  $\lambda_2 / \lambda_1$ .

Figure 3 in the previous section shows the DD residuals for both L1 and L2 which can separate the contribution of DD troposphere into the DD ionosphere delay. The only assumption behind this identification method is that the slope of  $\lambda_1 / \lambda_2$  purely follows the troposphere signature which may not be true in all environments due to many other effects, such as multipath, phase centre offset variations, etc. Figure 4 represents the geometry of the residuals of L1 and L2. Once the DD residual is determined at each epoch, it can be either within the troposphere identification boundary,  $\theta_2 - 7^\circ < P < \theta_2 + 7^\circ$  and  $\pi + (\theta_2 - 7^\circ) < P < \pi + (\theta_2 + 7^\circ)$  or outside the boundary limit.

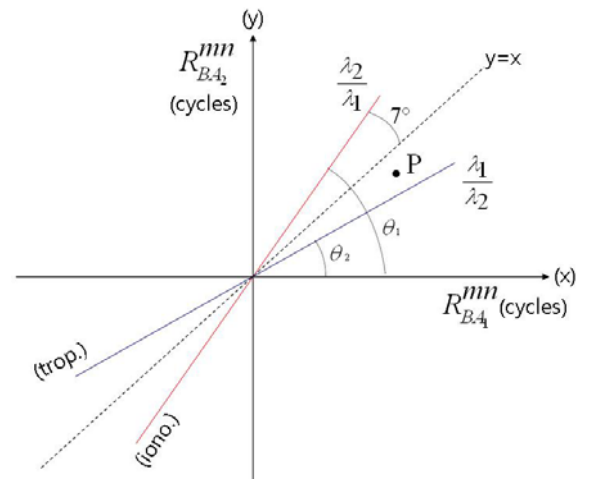


Figure 4. Identification of the contribution



Once the DD residual is within the boundary,  $\theta_2 - 7^\circ < P < \theta_2 + 7^\circ$  and  $\pi + (\theta_2 - 7^\circ) < P < \pi + (\theta_2 + 7^\circ)$ , the magnitude and the angle can be calculated. The magnitude can be used as a boundary determination, and the angle the region, using the simple mathematical relationship:

$$P_{mag} = \sqrt{(R_{BA_2}^{mn})^2 + (R_{BA_1}^{mn})^2} \quad (9)$$

$$\theta_p = \arctan(R_{BA_2}^{mn} / R_{BA_1}^{mn})$$

Figure 5 represents the identification and magnitude of the pair, PRN 19-27, during the anomaly based on the residual analysis. The magnitude presented is based on PPM. The top panel shows the magnitude of the residuals of L1 in terms of PPM versus elapsed time in seconds, and the middle panel represents the troposphere signature based on the L1 and L2 residual analysis described above. The lower panel is the ionosphere signature. Some of the missing points are not within the boundary limit which is not caused by either from the troposphere or the ionosphere. It clearly shows that most of the residuals follow the tropospheric contribution line and the magnitude in the slant range reaches almost 100 ppm level.

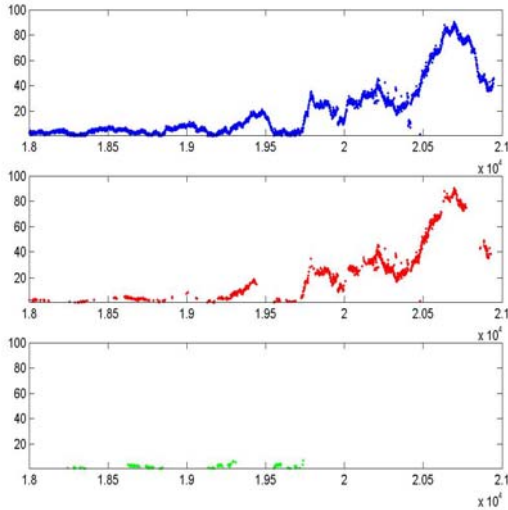


Figure 5. Identification of the contribution and magnitude in terms of ppm for an example pair, PRN19-27.

As expected, the residuals agree well with the DD tropospheric signature. During the anomaly, almost all other satellite pairs also showed similar patterns. At every epoch, the residual analysis can be performed to determine the magnitude of the troposphere. Once the magnitude is over the 20 ppm level in slant range,  $\Delta\tau$  is retrieved into zenith direction, and the value is applied to determine the weighting parameter  $\alpha$ .

$$\alpha_i = \frac{\Delta u}{(\Delta u + \Delta \tau_i)} \quad (10)$$

$$\Delta u = \zeta \cdot \bar{\alpha}_i, \quad i = 1, \dots, \text{number of satellite pairs}$$

The benefit of using this methodology lies in that fact that, as each satellite pair is examined to determine the weighting parameter in a different direction, it already contains the tropospheric gradient information as shown in Equation 10.

## DATA DESCRIPTION

The data which have a very strong localized tropospheric anomaly in Southern Texas on August 21, 2005 is reprocessed using the UNB-RTK platform. The baseline length is around 7.8 km, and we expected that the atmospheric effects should be highly correlated and thus easily eliminated in DD process. However, as is discussed, the DD residuals of the carrier-phase measurements reached around 100 ppm in slant range and most of the carrier-phase ambiguity resolutions on those specific periods failed to fix in UNB-RTK and in Bernese 5.0 as well. As expected, the errors created by mis-modeling of the troposphere in the least square adjustment are propagated into the vertical component. Figure 6 illustrates the image taken by infrared satellite and radar during the local anomaly in Southern Texas areas. We can clearly see a strong atmospheric effect in those regions. All of the data sets were recorded using the NovAtel™ OEM4 receiver with a data rate of 1 Hz. The observation time is almost 8 hours. The observation area is almost desert, and there are no buildings or trees that can usually cause multipath, cycle slips, etc. Therefore, we can presume that there are no significant contributions from the multipath in the data sets.

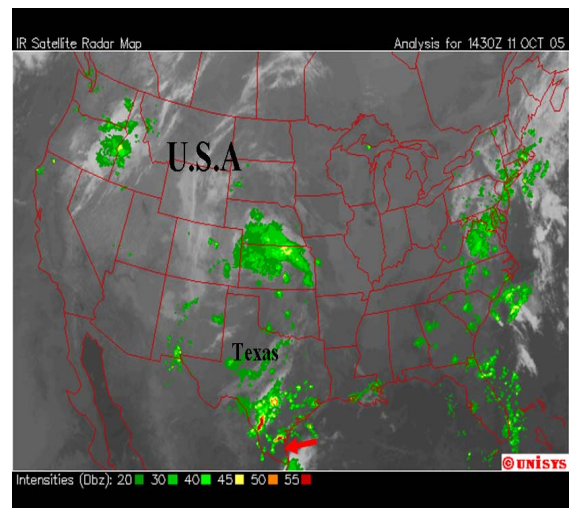


Figure 6. Satellite infrared image and radar map [UNISYS, 2005].

In this research, UNB-RTK software was used. Recently, the software has been enhanced for long-baseline applications, and for network applications. The software is based the OMEGA (Optimal Method for Estimating GPS Ambiguities) ambiguity search engine, and quality control algorithms described in Kim *et al.* [2003] and Kim and Langley [2005]. In addition, an optimal inter-frequency carrier-phase linear combination of the L1 and L2 measurements and receiver system noise estimation routine are the core of the software. The software carries out independent ambiguity resolution for the widelane, L1 and L2 observations to improve RTK positioning reliability.

## RESULTS

For processing the GPS data, the desired parameters are extracted from the software for further analysis using our methodology described earlier. Double differences are basic observables in this study. After correcting cycle slips, L1 (and L2) ambiguities were resolved using the OMEGA ambiguity search process. After all possible ambiguities were resolved, these ambiguities were introduced to obtain the final positioning solutions. During data processing, IGS final SP3 orbit products were used to mitigate the possible residual orbit errors. Once the double differences were reformulated, different  $\alpha$  values for each satellite pairs were applied for every epoch on the weighting scheme. Once the value of  $\alpha$  is selected using the selection criteria described above, the determined  $\alpha$  is used again to get the final positioning solution. The solution for the vertical component is discussed here. As is in Equation (6), if  $\alpha$  is determined based on the selection criteria and the actual magnitude of the wet delay to the zenith direction for each satellite pair, a new combined parameter ( $\zeta$ ) is estimated. Further, the determined weighting can be multiplied by  $\zeta$  in order to get the final vertical component of the baseline. Figure 7 represents a test result of a kinematic positioning solution for a typical estimation process. Due to the quality control criteria, the positioning solution of over 600 epochs during the anomaly can not be determined. For certain periods, the vertical component is over 20 cm as indicated in Figure 7. The poor epochs are probably due to the wrongly fixed cycle slips, or wrongly fixed ambiguities, due to the anomaly. Figure 8 illustrates the result of the vertical component when the new weighting scheme is applied. Although we could not get a dramatic vertical enhancement in this scenario, an improvement of around 20% in the vertical component was achieved in our initial test scenario. More interesting is that the anomaly which reaches at up to over 20 cm in Figure 7 does not exist in our processing results any more. Comparing the areas denoted by arrows in each figure, we can clearly see the improvement. Our preliminary results imply that our weighting scheme based on the residual analysis can be a good alternative for mitigating the unmodeled tropospheric delay.

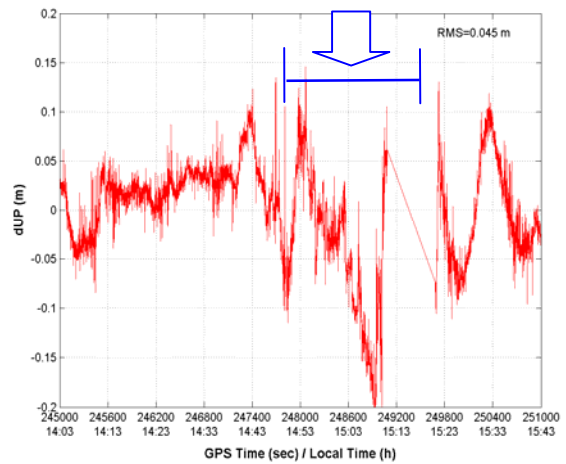


Figure 7. Original vertical component determined by UNB-RTK software during the anomaly period. The arrow represent the anomaly period.

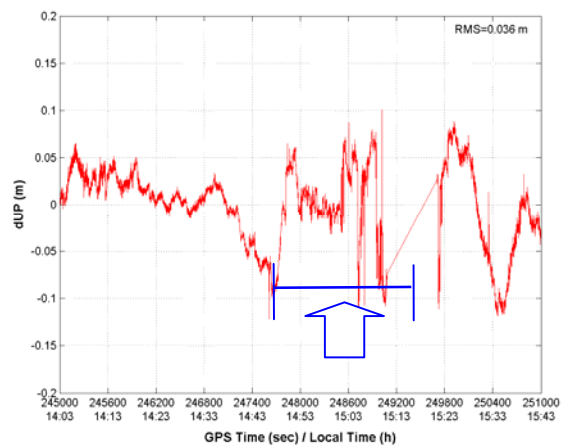


Figure 8. Vertical component determined by the proposed weighting scheme during the anomaly period. The arrow represent the anomaly period.

Even if most of the unmodeled tropospheric delay can be eliminated by a combined approach with proper weighting, some epochs do not show any improvement. We will further examine this issue after fully analyzing the best weighting parameter. For the purpose of the analysis, one of the ways of having the best weighting parameter may be from the criteria when the coordinate norm is minimal again. We can further compare the best set of weighting parameters from the criteria to the one applied herein to determine the optimal way of determining parameter weighting in different scenarios. Figure 9 presents the determined weighting parameter in this study. Lower values mean that the degree of the contribution of the troposphere was higher.

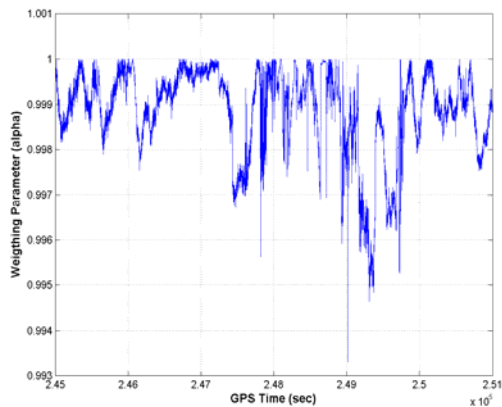


Figure 9. Weighting parameters. Lower values represent the higher tropospheric contribution.

## CONCLUSIONS AND COMMENTS

In short baselines with normal tropospheric conditions, as well as long baselines, the additional estimation of the tropospheric parameter may make the positioning solution unstable. This is because the introduction of another parameter in the estimation process may weaken the solution even though it is beneficial in reducing a certain type of bias or error. In order to avoid such difficulties, we introduced a new common parameter that combines the common zenith dependent parameters at our previous research. Once the combined parameter  $\zeta$  is determined based on the selected weighting parameter  $\alpha$ , the vertical component can be retrieved. Whenever  $\alpha$  is ‘properly’ chosen, our previous research showed that the corresponding coordinate solution can dramatically change and converge to the known positioning solution. The problem, however, lies in the selection of the  $\alpha$  coefficient which was somewhat arbitrary. Also, there is no strict numerical way to determine the  $\alpha$  coefficient. To avoid such difficulties, we determine  $\alpha$  when the residuals are decomposed into two different realms, either troposphere or ionosphere, and the calculated magnitude of the contribution of the troposphere for each satellite pair. This value is further used for determining the weighting parameter  $\alpha$ . By the use of this new method, we decorrelate the common zenith-dependent parameters which are the vertical component and the tropospheric parameter. Data from a severe localized tropospheric event were reprocessed to further analyze our proposed method. For the purpose of investigating the possible positioning improvement under severe imbalanced atmospheric conditions, relevant parameters from UNB-RTK were reprocessed. We investigated the possible positioning improvements after introducing the combined parameter in the processing of the data collected during severe inhomogeneous tropospheric conditions. From previous research, the local troposphere anomaly is highly correlated

with the vertical component. Therefore, improper mitigation of that effect can severely degrade the vertical positioning performance, even for short baselines.

By introducing the methodology of the new weighting scheme to de-correlate the vertical and tropospheric parameters for the purpose of mitigating the mis-modeled tropospheric delay, positioning improvement can be achieved in 20% in the vertical component. In addition, degradation of the vertical component during the anomaly period in the conventional approach is almost disappeared. This may mean that our approach is resistant to the anomalous tropospheric case even if there are more challenges to be solved in our approach.

Even if most of the unmodeled tropospheric delay can be eliminated by our combined approach with proper weighting, some epochs do not show any improvement. We will further examine this issue after fully analyzing the best weighting parameter. One possibility is reverse analysis based on the best set of weighting parameters under norm of coordinates; we can further analyze the severe tropospheric anomaly effects by simulated data sets in order to have an optimal choice of the parameter.

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