Quality Control Techniques and Issues in GPS Applications: Stochastic Modeling and Reliability Test

Donghyun Kim and Richard B. Langley

GPS Research Group
Geodetic Research Laboratory, Department of Geodesy and Geomatics Engineering,
University of New Brunswick, Fredericton, N.B., Canada E3B 5A3
kim@unb.ca, lang@unb.ca

ABSTRACT

This paper summarizes a new quality control method including a new cycle-slip correction method which enables instantaneous correction (*i.e.*, using only current epoch's GPS carrier-phase measurements) at the data quality control stage and a new approach for the stochastic model for kinematic GPS positioning which can be derived directly from the observation time series under a simple assumption. The method was originally developed for real-time kinematic applications which require high integrity and consistent high-precision positioning results with the carrier-phase measurements.

INTRODUCTION

In order to attain consistent high-precision positioning results with GPS carrier-phase measurements, errors unspecified in a functional or stochastic model (errors of omission) must be correctly detected and removed or otherwise handled at the data processing stage. Such errors in the carrier-phase measurements may include cycle slips, receiver clock jumps, multipath, diffraction, ionospheric scintillation, *etc*. Reliability, which refers to the ability to detect such errors and to estimate the effects that they may have on a solution, is one of the main issues in quality control. Texts containing detailed discussions of this topic include those by *Leick* [1995] and *Teunissen and Kleusberg* [1998]. A comprehensive investigation of quality issues in real-time GPS positioning has been carried out by the Special Study Group (SSG) 1.154 of the International Association of Geodesy (IAG) during 1996-1999 [*Rizos*, 1999].

The effects of cycle slips and receiver clock jumps can be easily captured either in the measurement or parameter domain due to their systematic characteristics. Their systematic effects on the carrier-phase measurements can be almost completely removed once they are correctly detected and identified. On the other hand, multipath, diffraction, ionospheric scintillation, *etc*. have temporal and spatial characteristics which are more or less quasi-random. These quasi-random errors cannot be completely eliminated and must be handled using a rigorous mathematical approach such as data snooping theory [*Baarda*, 1968]. However, statistical testing and reliability analysis can only be efficient if the stochastic models are correctly known or well approximated.

In general, the stochastic model is involved in three stages of the GPS data processing – the quality control of the measurements, the ambiguity resolution, and the least-squares estimation. If the stochastic model is not correct, the quality control process used to detect and fix cycle slips (and errors) in L1 and L2 carrier-phase observations may not work correctly. The result of faulty cycle-slip fixing can be a disaster in the applications using GPS carrier-phase observations because it introduces artificial biases into the observations and subsequently, the estimated parameter values. An incorrect stochastic model also makes it difficult to resolve correct ambiguities. If the resolved ambiguities are not correct, similar effects to the faulty cycle-slip fixing are transferred to the parameter values. Compared with the effect of the faulty cycle-slip fixing and incorrect ambiguities on least-squares solutions, that of an incorrect stochastic model is less important. However, the quality of the solutions can be interpreted as too optimistic or conservative if an incorrect stochastic model is used.

As has been experienced, quasi-random errors (multipath, diffraction, ionospheric scintillation, *etc*.) are often mixed with systematic ones (cycle slips and receiver clock jumps) in real world situations. One reasonable approach for handling errors in such situations is to separate the systematic ones from the quasi-random ones. Estimating the quasi-random errors after removing the systematic ones can provide more reliable results in terms of least-squares estimation. This is the main idea

Presented at the 2001 International Symposium on GPS/GNSS (the 8th GNSS Workshop), Jeju Island, Korea, 7-9 November 2001

implemented in our quality control algorithm including cycle-slip correction.

CORRELATION IN THE OBSERVATION TIME SERIES

When we talk about the stochastic model, we are usually interested in a fully populated variance-covariance matrix. To obtain this matrix, we have to take into account correlation in the observation time series. Three typical physical correlation types exist in the GPS observations: First, cross-correlation which represents the correlation between different observation types (*e.g.*, L1 and L2 carrier-phase, and C1, P1 and P2 pseudorange). Second, temporal correlation which represents the correlation between sequential observations (epoch to epoch). Third, spatial correlation which represents the correlation among 'all-in-view' simultaneous observations. Another possible physical correlation is inter-channel correlation which may exist among the receiver channels. If we use a differencing scheme in the measurement domain, mathematical correlation also exists.

It may be helpful to classify the correlation types in terms of correlation sources. The temporal and spatial correlations are due to biases such as atmospheric effects (*i.e.*, ionosphere and troposphere), satellite orbit bias, multipath and so on. On the other hand, the cross-correlation and inter-channel correlation are due to the receiver signal processing methods.

As we usually experience, correlation makes it difficult to obtain a correct stochastic model. However, correlation is not always so bad because some biases can be cancelled due to correlation in the observation time series. For example, spatially correlated biases can be cancelled by differencing the observations in the measurement domain. The single- and double-difference are such cases. Applying the same concept, temporally correlated biases can be cancelled by differencing the observations in the time domain (*e.g.*, triple-, quadruple- and quintuple-difference). We take advantage of these concepts in estimating the stochastic model in our approach.

ANOMALOUS OBSERVATION DATA

The quality of GPS positioning is dependent on a number of factors. To attain high-precision positioning results, we need to identify the main error sources impacting on the quality of the measurements. In terms of data processing, receiver clock jumps, cycle slips and quasi-random errors are the main sources which can deteriorate the quality of the measurements and subsequently, the quality of positioning results.

Receiver Clock Jumps

Most receivers attempt to keep their internal clocks synchronized to GPS Time. This is done by periodically adjusting the clock by inserting time jumps. The actual mechanism of how the clock in a particular manufacturer's receiver is adjusted is typically proprietary. However, like cycle slips, their effects on the code and phase observables are more or less known to users and hence it is possible to remove their effects almost completely. At the moment of the clock correction, two main effects are transferred into the code and phase observables as illustrated in the following equation:

$$\Psi\left(t+\Delta\right) \approx \Psi\left(t\right) + \dot{\Psi}\cdot\Delta \approx \Psi\left(t\right) + \dot{\rho}\cdot\Delta - c\cdot\Delta\,, \qquad (1)$$

where Ψ represents the carrier-phase (or pseudorange) measurements (in distance units) and $\dot{\Psi}$ its time rate of change; Δ is the clock jump (in time units) which is assumed to be a very small value (*e.g.*, less than or equal to one millisecond); $\dot{\rho}$ is the geometric satellite-receiver range rate (in distance per time units); and c is the vacuum speed of light. The effects of the geometric range corresponding to the clock jump ($c\cdot\Delta$) are common to all measurements at the moment of the clock jump at one receiver. By single-differencing (SD) the measurements between satellites or double-differencing (DD) the measurements (that is, differencing between receivers followed by differencing between satellites or vice versa), the common effects can be removed. However, the effects of the geometric range rate corresponding to the jump ($\dot{\rho}\cdot\Delta$) are different for each observation. They cannot be removed by the SD operation. Instead, Doppler frequency or carrier-phase difference measurements should be used for correcting such effects. The clock jumps can be easily detected in both the measurement and parameter domains.

Two typical examples of receiver clock jumps are illustrated in Figures 1 and 2. The first example is a millisecond jump which occurred in the Ashtech Z-12 as seen in Figure 1. The top panel shows the receiver clock offsets estimated by the C/A-code pseudoranges and reveals that the millisecond jumps occurred about every 7 minutes. As was explained previously, the effects on the geometric range corresponding to the jumps are removed in the DD measurements. However, the effects on the geometric range rate corresponding to the jumps still remain in the DD measurements. The spikes in the TD time series, as seen in the middle and bottom panels, disclose the significance of such effects. Using the Doppler frequency, such effects can be easily estimated and removed. The second example is a very small clock jump which occurred in the Navcom NCT-2000D as seen in Figure 2. The top panel shows the receiver clock offset estimates including the microsecond-level jumps which occur every second. Since the jumps are so small, the effects of the geometric range rate corresponding to the jumps seem to be buried in the noise of the TD measurements; *i.e.*, the TD time series would not reveal the spikes as verified in the middle and bottom panels. However, it should be noted that such effects could reach the level of a few centimetres when a satellite is close to the horizon and at the same time the receiver experiences high dynamics. Without correcting for such effects, we may have difficulty in fixing cycle slips and subsequently, in overall quality control of the carrier-phase measurements.

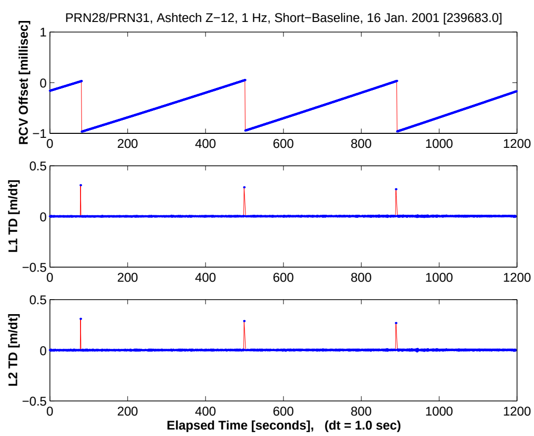

Fig. 1 – Effects of the receiver clock jumps (Ashtech Z-12): the receiver clock offset estimates (top); and the L1 and L2 TD measurements disclosing the effects of the geometric range rate corresponding to the jumps (middle and bottom). The number in square brackets is the GPS Time of first observation. Vertical axis scale for TD values is normalized using the observation sampling interval (dt).

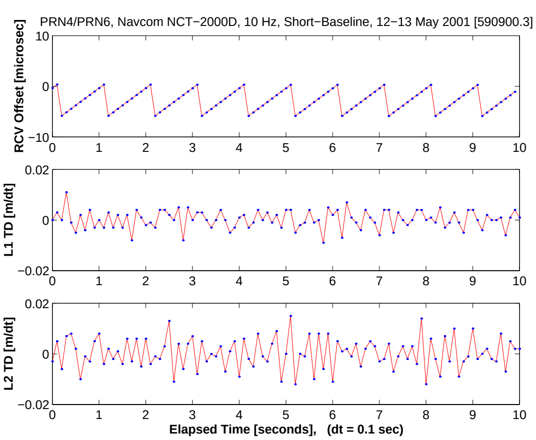

Fig. 2 – Effects of the receiver clock jumps (Navcom NCT-2000D): the receiver clock offset estimates (top); and the L1 and L2 TD measurements hiding the effects of the geometric range rate corresponding to the jumps (middle and bottom).

Quasi-random Errors

Multipath, diffraction, ionospheric scintillation, *etc.* may be the main sources of quasi-random errors, which are apt to be omitted in the functional and stochastic models. Since least-squares estimation, when errors are present, tends to hide (reduce) their impact and distribute their effects throughout the entire set of measurements, it is better to handle cycle slips and clock jumps separately from quasi-random errors. To detect and remove quasi-random errors, we have to test a null hypothesis (that is, no errors in the measurements) against an alternative hypothesis which describes the type of mis-specifications in the models.

Quasi-random errors usually make it difficult to fix cycle slips correctly. Since the cycle slip constraint of discontinuities of an integer number of cycles does not apply to quasi-random errors, fixing cycle slips is a big challenge when systematic and quasi-random errors are interspersed. One typical example of signal diffraction due to obstructions is illustrated in Figures 3 through 5. Figure 3 shows that one satellite (PRN 21) was temporarily (for about 2 minutes) blocked by the penthouse on the roof of Gillin Hall at the University of New Brunswick. (Head Hall data acquired during the period of obstruction on Gillin Hall has been deleted in the bottom panel) Rapid signal degradation at the moment of signal obstruction as seen in the middle panel implies that there could be significant deterioration in the quality of the measurements. On the other hand, Figure 4 shows that the other satellite (PRN 27), which was used with PRN 21 in generating the DD measurements, did not suffer from such effects. This appears to confirm that the spikes in the TD measurements seen in Figure 5 were due to signal diffraction.

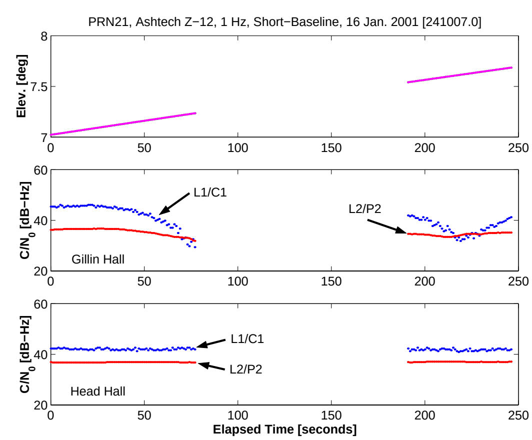

Fig. 3 – Signal degradation due to diffraction (middle).

a failure in the receiver software which leads to incorrect signal processing.

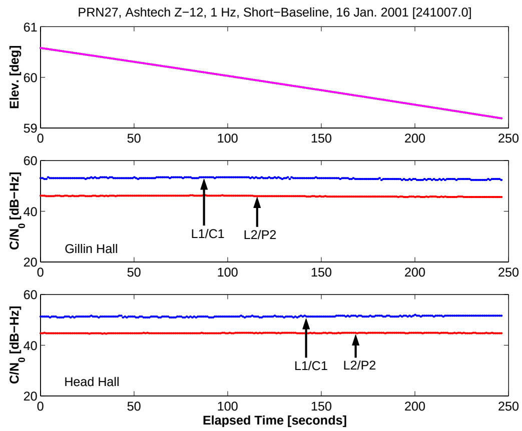

Fig. 4 – Normal (i.e., not problematic) signal reception situations confirmed by the satellite elevation angles and C/N_0 values.

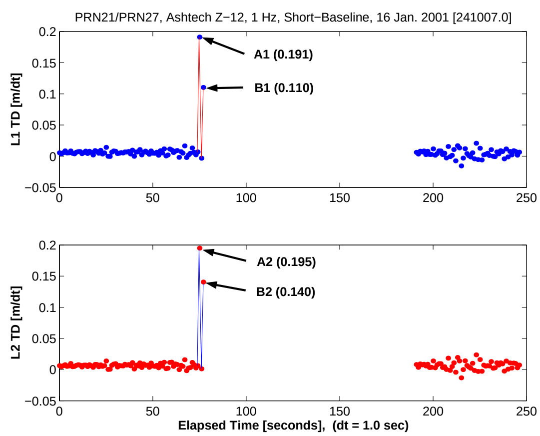

Fig. 5 – Effects of signal diffraction in the L1 and L2 TD measurements.

Cycle Slips

Cycle slips are discontinuities of an integer number of cycles in the measured (integrated) carrier phase resulting from a temporary loss-of-lock in the carrier tracking loop of a GPS receiver. In this event, the integer counter is re-initialized which causes a jump in the instantaneous accumulated phase by an integer number of cycles. Three causes of cycle slips can be distinguished [*Hofmann-Wellenhof et al.*, 1999]: First, cycle slips are caused by obstructions of the satellite signal due to trees, buildings, bridges, mountains, *etc*. The second cause of cycle slips is a low signal-to-noise ratio (SNR) or alternatively carrier-to-noise-power-density ratio (C/N_0) due to severe ionospheric conditions, multipath, high receiver dynamics, or low satellite elevation angle. A third cause is a failure in the receiver software which leads to incorrect signal processing.

Cycle slips in the phase data must be corrected to utilize the full measurement strength of the phase observable. The process of cycle-slip correction involves detecting the slip, estimating the exact number of L1 and L2 frequency cycles that comprise the slip, and actually correcting the phase measurements by these integer estimates. Cycle slip detection and correction requires the location of the jump and the determination of its size. It can be completely removed once it is correctly detected and identified.

PREVIOUS WORK ON STOCHASTIC MODELING

A brief review in terms of advantages and disadvantages of the previous work on stochastic modeling which has been carried out by many research groups all over the world will be useful in figuring out the problems related to particular GPS applications such as short-baseline or long-baseline situations, static or kinematic mode, and real-time or post-processing operations. There are several approaches which provide somewhat realistic stochastic models: the elevation-angle dependent function approach [*Euler and Goad*, 1991; *Jin*, 1996]; the SNR or alternatively C/N_0 approach [*Hartinger and Brunner*, 1998; *Barnes et al.*, 1998; *Collins and Langley*, 1999]; the least-squares adaptation approach [*Han*, 1997; *Wang et al.*, 1998; *Wang*, 1999; *Tiberius and Kenselaar*, 2000]. Fundamental discussions on the observation noise were given by *Langley* [1997] and *Tiberius et al.* [1999].

The elevation-angle dependent function approach takes into account the elevation-angle dependence of the observation noise. This approach is easy to implement in data processing software as long as the parameters of the function are calibrated in the laboratory. However, this approach does not provide information on cross-correlation and spatial correlation. This means that we cannot get the fully populated variance-covariance matrix from the approach. Furthermore, it should be noted that the elevation-angle dependence of the observation noise often varies with the particular kinematic situation. The elevation-angle dependence of the observation noise is induced mainly by the receiver antenna's gain pattern, with other factors such as atmospheric signal attenuation (spacecraft antenna beam-shaping ensures an almost uniform signal field strength independent of elevation angle). The elevation angle is normally computed with respect to the local geodetic horizon plane at the antenna phase center regardless of the actual orientation of the antenna. Accordingly, the relationship between antenna gain and the signal elevation angle may be difficult to assert when the antenna orientation is changing which can happen often in kinematic situations.

The SNR (or alternatively C/N_0) approach provides actual observation noise information which can be derived directly from measurements of the quality of each pseudorange and carrier-phase observation. This information is contained in the SNR measurement. This value determines, in part, how well the receiver's tracking loops can track the signals and hence (to a large degree) how precisely the receiver obtains pseudorange and carrier-phase observations [*Langley*, 1997]. Since multipath signals can adversely impact the receiver SNR depending on whether the direct and reflected signal components reaching the receiver combine constructively or destructively [*Cox et al.*, 1999], this approach does provide realistic observation noise in strong multipath environments. This may be a good approach for the precise point positioning (PPP) technique in which there is not enough redundancy to mitigate the effect of multipath except by down-weighting. Although some GPS receiver manufacturers provide SNR values in their data streams, meaningful SNR values are not always easy to come by (see *Collins and Langley* [1999]). Furthermore, this approach also does not provide the fully populated variance-covariance matrix.

The least-squares adaptation approach, which can be incorporated within a recursive processing scheme such as the Kalman filter and the sequential least-squares estimator, generally provides optimal stochastic models. In addition, it is easy to obtain information for the cross-correlation and spatial correlation from the approach. However, the optimality of the least-squares estimation is not always guaranteed because it depends on assumptions on correlation: *e.g.*, no temporal correlation or a certain order of temporal correlation. The estimates can be too optimistic or conservative if the assumptions are not satisfied. Furthermore, there are particular applications where no observation redundancy is provided. In long-baseline applications, typically we cannot get observation redundancy as long as we take into account all significant biases in the functional model. In such situations, the least-squares adaptation approach will not work.

OBSERVATION NOISE ESTIMATION USING DIFFERENCING IN THE TIME DOMAIN

Basically, we try to overcome the three main problems of the existing approaches (*i.e.*, lack of a fully populated variance-covariance matrix, missing temporal correlation, and no observation redundancy in long-baseline applications) in our approach. We assume that DD observation time series are given. Hereafter, we will leave out the notation of DD (or $\nabla\Delta$) for the observations, biases and errors. In short-baseline situations, the effects of the (correlated) biases are usually ignorable. Accordingly, temporal correlation is ignorable in estimating the stochastic model because temporal correlation reflects mainly the behaviour of the biases (although observation smoothing by the receiver will introduce some temporal correlation in observation noise). However, it should be noted that the effect of the double-differenced multipath is not always ignorable even in short-baseline situations. In other words, temporal correlation may exist depending on multipath environments. In long-baseline situations, the biases are not ignorable, so that temporal correlation usually exists in the observation time series.

To remove the non-random behaviour of the observation time series, we use a differencing scheme in the time domain including the triple-difference (TD; differencing consecutive observations after deleting cycle-slip spikes), quadruple-difference (QD; differencing consecutive TD observations), quintuple-difference (dQD; differencing consecutive QD observations), and so on. In this approach, we assume that the effects of any biases can be canceled in the differencing process, so that only the effect of observation noise (assumed as white noise) remains in the resulting time series. This assumption can be justified as long as we can obtain time series with a sufficiently short sampling interval. If the observation time series samples are obtained with a smaller time interval (*i.e.*, a higher data rate) than the time constant of each component of the biases, the assumption can be easily satisfied. This reasoning is based on the fact that differences are generated by subtractive filters. These are high-pass filters damping low frequencies and eliminating constant components. High frequency components are amplified.

There exist two degrees of freedom in our approach: *i.e.*, the order of the differencing and the data rate must be determined in terms of optimality. In general, increasing the order of the differencing continuously is pointless because the time-correlated biases are easily canceled at a low order. The dQD differencing is sufficient for almost all situations according to our analyses. On the other hand, the optimal data rate is usually dependent on particular applications such as static, low-dynamics kinematic, and high-dynamics kinematic applications. Although determining the optimal data rate is more or less arbitrary, there is a general rule which can be understood in terms of the physics inherent in the differencing process: *i.e.*, the data rate should be high enough to make each component of the biases temporally correlated. If the effects of high-frequency (compared with the data rate) components of the biases are significant in the time series, the data rate should be increased to cancel the effects of such components in the differencing process. Most problematic high-frequency component in our approach is the jerk of the geometric range due to moving platform dynamics. Taking a look at the problem in terms of a numerical process, we can get a better insight into how to come up with a solution to the problem. For example, consider the L1 carrier-phase dQD observable:

$$\dddot{\Phi}_1 = \dddot{\rho} + \dddot{\tau} + \dddot{s} - \dddot{I} + \dddot{b}_1 + \dddot{n}_1 + \dddot{\varepsilon}_1 , \tag{2}$$

where Φ_1 stands for the double-difference L1 observable; ρ for the geometric range; s for the satellite orbit bias; τ for the tropospheric delay; I for the L1 ionospheric delay; b_1 for multipath in L1 carrier phase; n for the ambiguities (in distance units); and ε_1 for observation noise of the L1 carrier phases. Using the one-dimensional Taylor series including higher-order time derivatives for each of the biases, we have

$$\begin{aligned} S(t) = S(t_0) + S'(t_0)(t-t_0) + \tfrac{1}{2} S''(t_0)(t-t_0)^2 \\ + \tfrac{1}{6} S'''(t_0)(t-t_0)^3 + R(t), \end{aligned} \tag{3}$$

where S represents each biases and R is a remainder term known as the Lagrange remainder. Assuming that the observation time interval is $\delta(=t-t_0)$, we have the following dQD observable:

$$\begin{aligned} \dddot{S}(t_3) &= S(t_3) - 3 \cdot S(t_2) + 3 \cdot S(t_1) - S(t_0) \\ &= S'''(t_0)\delta^3 + \Sigma_R(t_3) , \end{aligned} \tag{4}$$

where $\Sigma_R(t_3)$ is the effect of dQD for the remainder R. Substituting Eq. (4) into (2) gives

$$\dddot{\Phi}_1(t_3) = \left[\sum_{\forall S} S'''(t_0) \right] \delta^3 + \sum_{\forall S} \left[\Sigma_R(t_3) \right] + \dddot{\varepsilon}_1(t_3) , \tag{5}$$

where

$$\sum_{\forall S} S'''(t_0) = \left[\rho''' + \tau''' + s''' - I''' + b_1''' + n_1''' \right](t_0) . \tag{6}$$

Eqs. (5) and (6) clearly show the relationship between the high-frequency components and the data rate in the differencing process. If the effects of the high-frequency components in the right-hand side of Eq. (6) are small enough to be ignorable and/or the data rate (1/δ) is high in Eq. (5), and if the effect of the second term in the right-hand side of Eq. (5) (i.e., the effect of dQD for the remainder R) is also small enough to be ignorable, we can get an acceptable inference as:

$$\dddot{\Phi}_1 \approx \dddot{\varepsilon}_1 . \tag{7}$$

As long as the differencing process satisfies Eq. (7), the fully populated variance-covariance matrix for the dQD observation noise $\dddot{\varepsilon}_1$ can be easily estimated using n dQD samples for each double-difference satellite pair time series:

$$\hat{\mathbf{Q}}_{\mathbf{dQD}} = \operatorname{cov}\left[\mathbf{dQD} \right], \tag{8}$$

where $\mathbf{dQD} = \left[dQD_{i,j} \right]$; subscript i stands for a sample number (epoch); subscript j identifies a particular double-difference satellite pair time series number; and $\operatorname{cov}[\cdot]$ is the variance-covariance operator. Since there exists a certain relationship between the original time series and the dQD time series as shown in Eq. (4), assuming that four consecutive L1 observations have the same variance, the fully populated variance-covariance matrix for the observation noise also can be estimated:

$$\hat{\mathbf{Q}}_{\mathbf{DD}} = \frac{1}{20} \hat{\mathbf{Q}}_{\mathbf{dQD}} . \tag{9}$$

Note that the size of samples n should be determined in terms of the unbiasedness of the estimates. In general, the larger the size of samples, the better the estimates. However, there may be a trade-off to some degree in real-time implementation.

We have justified our approach using test data sets obtained in a variety of situations [*Kim and Langley*, 2001a]. Based on our justification, we estimated fully populated variance-covariance matrices for observation noise and compared the results with those of the C/N$_0$ approach. A typical example is illustrated in Figure 6.

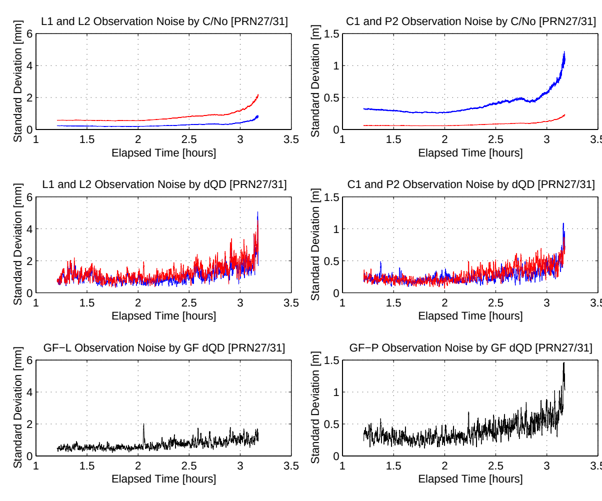

Fig 6 – Observation noise estimation: The red line is the estimates for L1(C1) and the blue line for L2(P2).

Several comments should be given to understand observation noise estimation figures: First, when we estimate observation noise using the C/N_0 approach (i.e., the top two plots in each figure), we must know at least two factors correctly – a conversion equation between the receiver's SNR output and the C/N_0, and the signal tracking loop bandwidths. We used the published SNR-to-C/N_0 conversion equation for the Ashtech Z-12 receiver. However, we did not have available the correct information for the signal tracking loop bandwidths. Therefore, we used an example value (the same value for L1, L2, C1 and P2) given in *Langley* [1997]. The effect of incorrect values for signal tracking loop bandwidths, however, just scales the true values. This means that the pattern of the original estimates holds. Second, to justify the effect of platform dynamics, we estimated observation noise using the time series of the geometry-free (GF) linear combinations for carrier-phase and pseudorange (i.e., the bottom two plots in Figure 6), which are free from platform dynamics.

QUALITY CONTROL WITH AN INSTANTANEOUS CYCLE-SLIP CORRECTION TECHNIQUE

One of the various methods for detecting and identifying cycle slips is to obtain the TD measurements of carrier phases first. By triple-differencing the measurements, biases such as the clock offsets of the receivers and GPS satellites, and ambiguities can be removed. In most GPS applications, regardless of surveying mode (static or kinematic) and baseline length (short, medium or long), the effects of the triple-differenced biases and noise (*i.e.*, atmospheric delay, satellite orbit bias, multipath, and receiver system noise) are more or less below a few centimetres in size as long as the observation sampling interval is relatively short (*e.g.*, sampling interval less than 1 minute). There could be exceptional situations such as an ionospheric disturbance, extremely long baselines, and huge (rapid) variation of the heights of surveying points in which the combined effects of the biases and noise can exceed the wavelengths of the L1 and L2 carrier phases. However, such situations can be easily controlled through adjusting the sampling rate so that the combined effects of the biases and noise can be reduced to a level below a few centimetres.

When the estimates of the TD geometric ranges are available either from the Doppler frequency or TD pseudoranges, we can obtain reliable cycle-slip candidates using the first two moments of the TD carrier-phase prediction residuals (that is, differences between the TD carrier-phase measurements and the estimates of the TD geometric ranges). When dual-frequency carrier phases are available, we can reduce, to a large extent, the number of cycle-slip candidates using the TD geometry-free phase (a scaled version of which is called the differential ionospheric delay rate).

Cycle-Slip Validation

Fixing cycle slips in the TD measurements is conceptually the same problem as resolving ambiguities in the DD measurements. Consider the linearized model of the TD observables:

$$\begin{aligned} &\mathbf{y} = \mathbf{Ac} + \mathbf{Bx} + \mathbf{e}, \quad \mathbf{c} \in \mathbb{Z}^n, \quad \mathbf{x} \in \mathbb{R}^u \\ &Cov[\mathbf{y}] = \mathbf{Q}, \end{aligned} \tag{10}$$

where y is the $n \times 1$ vector of the differences between the TD measurements and their computed values; n is the number of measurements; c is the $n \times 1$ vector of the cycle-slip candidates; x is the $u \times 1$ vector of all other unknown parameters including position and other parameters of interest; u is the number of all other unknowns except cycle slips; A and B are the design matrices of the cycle-slip candidates and the other unknown parameters; e is the $n \times 1$ vector of the random errors. The first step for cycle-slip validation is to search for the best and second best cycle-slip candidates which minimize the quadratic form of the residuals. The residuals of least-squares estimation for cycle-slip candidates are given as:

$$\hat{\mathbf{v}} = \mathbf{y}' - \mathbf{B}\hat{\mathbf{x}}, \tag{11}$$

where

$$\begin{aligned} &\mathbf{y}' = \mathbf{y} - \mathbf{Ac} \\ &\hat{\mathbf{x}} = \left(\mathbf{B}^{\mathbf{T}}\mathbf{Q}^{\mathbf{-1}}\mathbf{B}\right)^{\mathbf{-1}} \mathbf{B}^{\mathbf{T}}\mathbf{Q}^{\mathbf{-1}}\mathbf{y}'. \end{aligned} \tag{12}$$

Then, discrimination power between two candidates is measured by comparing their likelihood. We follow a conventional discrimination test procedure similar to that described by Wang *et al*. [1998]. A test statistic for cycle-slip validation is given by

$$d = \Omega_{\mathbf{c1}} - \Omega_{\mathbf{c2}}, \tag{13}$$

where $\Omega_{\mathbf{c1}}$ and $\Omega_{\mathbf{c2}}$ are the quadratic form of the residuals of the best and second best candidates. A statistical test is performed using the following null and alternative hypotheses:

$$H_0 : E[d] = 0, \quad H_1 : E[d] \neq 0. \tag{14}$$

A test statistic for testing the above hypotheses is given by

$$W = \frac{d}{\sqrt{Cov(d)}}, \tag{15}$$

where

$$\begin{aligned} Cov[d] &= 4(\mathbf{c}_1 - \mathbf{c}_2)^{\mathrm{T}} \mathbf{Q}_{\mathbf{c}}^{-1} (\mathbf{c}_1 - \mathbf{c}_2) \\ \mathbf{Q}_{\mathbf{c}}^{-1} &= \mathbf{Q}^{-1} - \mathbf{Q}^{-1}\mathbf{B}(\mathbf{B}^{\mathrm{T}}\mathbf{Q}^{-1}\mathbf{B})^{-1}\mathbf{B}^{\mathrm{T}}\mathbf{Q}^{-1}. \end{aligned} \tag{16}$$

If y is assumed as having a normal distribution, d is normally distributed. Therefore, W has mean 0 and standard deviation 1 under the null hypothesis. Adopting a confidence level α, it will be declared that the likelihood of the best cycle-slip candidate is significantly larger than that of the second best one if

$$W > N(0,1;1-\alpha). \tag{17}$$

Finally, a reliability test is carried out after fixing cycle slips in order to diagnose whether errors still remain in the measurements. Figure 7 depicts our quality control algorithm including cycle-slip correction.

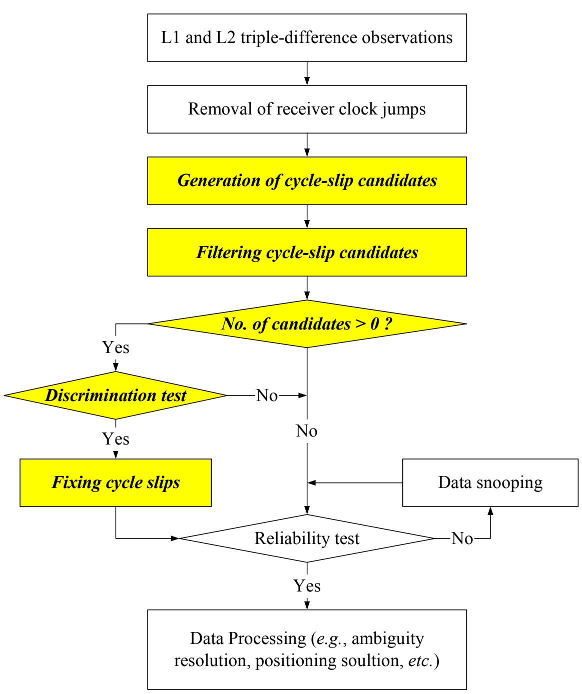

Fig. 7 – Quality control of the carrier-phase measurements in conjunction with an instantaneous cycle-slip correction routine (steps labelled in italics).

We have tested our approach using data sets recorded in a variety of situations [*Kim and Langley*, 2001b]. Two aspects have stood out in the tests. First, the approach is capable of instantaneous cycle-slip correction. This means that it is suitable for real-time applications, not only for post-processing. Second, the approach can handle cycle slips in low quality measurements. Unlike conventional approaches, therefore, we do not have to discard the measurements obtained at low elevation angles and from weak signals with low C/N_0 values. As a result, our approach tends to increase observation redundancy and hence system performance (*e.g.*, integrity, continuity, accuracy and availability) is improved. A typical example which shows the performance of our approach is illustrated in Figures 8 through 10. Figures 8 and 9 show the two satellites which were used in generating the DD measurements. They were observed more or less at high elevation angles. However, the signal of one satellite (PRN 8) was heavily affected by what appears to be ground-bounce multipath as seen in Figure 8. While the L1 signal was still fairly strong in such situations, the L2 signal was fading quickly and automatic gain control attempted to compensate for the low gain situation. As a result, we can see L2/P2 C/N_0 gain jumps in the middle and bottom panels. Figure 10 shows that the L1 and L2 TD measurements were contaminated by many cycle slips (the first and third panels) and confirms that our approach fixed them perfectly (the second and fourth panels).

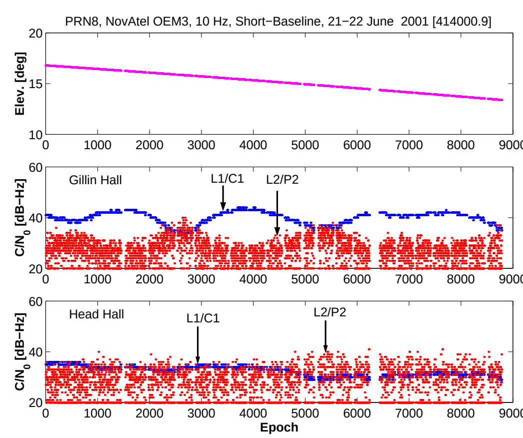

Fig. 8 – Signal degradation due to multipath.

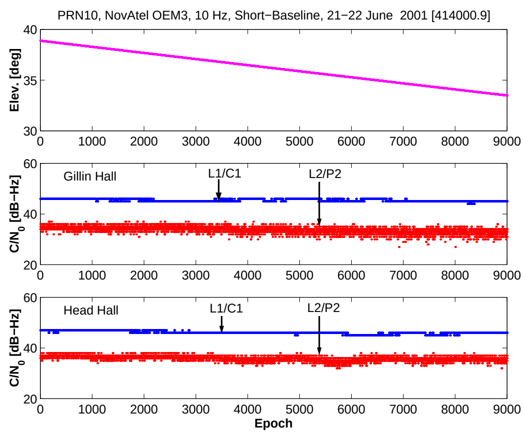

Fig. 9 – Normal (i.e., not problematic) signal reception situations confirmed by the satellite elevation angles and C/N_0 values.

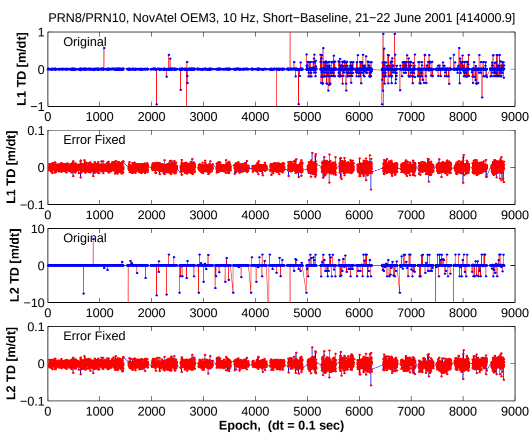

Fig. 10 – Performance of instantaneous cycle-slip correction.

CONCLUSIONS

Over the past decade, a number of methods have been developed to handle errors in the carrier-phase measurements. There are, in large, two main research streams in this area: cycle-slip-related research and quality-control-related research. The former focuses mainly on cycle slips and takes advantage of the systematic characteristics of cycle slips, more or less ignoring the effects of the other errors. As a matter of fact, cycle slips are the biggest error source if they remain in the carrier-phase measurements. On the other hand, the latter approach considers that all biases and errors must be detected by a rigorous statistical test such as the reliability test. This approach tends more or less not to utilize the advantage taken by the former. We use a hybrid method for quality control: systematic errors such as cycle slips and receiver clock jumps are examined and corrected first; then, a reliability test is carried out to reduce the effects of quasi-random errors.

Three main features differentiate our approach from conventional techniques. First, our approach is suitable for kinematic applications either in real-time or in post-processing mode. Unlike conventional approaches such as a sequential least-squares estimator or Kalman filter which uses the prediction values of the measurements for quality control, our approach does not require them. Instead, it utilizes only the current epoch's measurements for quality control. Therefore, the approach can attain high performance even when a receiver platform is maneuvering. Second, the approach can handle cycle slips in low quality measurements, so that we do not have to discard the measurements obtained at low elevation angles and from weak signals with low C/N_0 values. As a result, the approach tends to increase observation redundancy and hence system performance in terms of integrity, continuity, accuracy and availability is improved. Third, the approach enables us to determine system performance by simply assigning the confidence level of the parameters for cycle-slip candidates when designing the system.

Tests carried out in a variety of situations including static and kinematic modes, short-baseline and long-baseline situations, and low and high data rates have confirmed the high performance of our approach. A worst-case simulation test has also proved its performance. However, we are aware that the same generic (intrinsic) limitations as with least-squares estimation still remain in our approach; *i.e.*, the need for redundancy and stochastic modelling. To increase redundancy, we need to use as many measurements as are available in real world scenarios. In this case, many problematic situations can occur in the measurements, particularly ones obtained at low elevation angles and from weak signals with low C/N_0 values. As mentioned previously, our approach indeed works well in such situations. Finally, statistical testing and reliability analysis can only be efficient if the stochastic models are correctly known or well approximated. obtaining a reliable stochastic model is still a big challenge. However, the "differencing-in-time" method has been successfully tested and implemented in our approach, in which we have tried to overcome the three main problems of existing approaches in determining the stochastic model for GPS observations – lack of a fully populated variance-covariance matrix, missing temporal correlation, and no observation redundancy in long-baseline applications.

ACKNOWLEDGEMENTS

The research reported in this paper was conducted under the program of the GEOIDE Network of Centres of Excellence (Project ENV#14). The support of the Canadian Coast Guard; the Canadian Hydrographic Service; VIASAT Géo-Technologie Inc.; Geomatics Canada; NovAtel Inc.; and the Centre de Recherche en Géomatique, Université Laval is gratefully acknowledged.

This paper is based, in part, on *Kim and Langley* [2001a] presented at the Institute of Navigation National Technical Meeting, held in Long Beach, California, U.S.A. in January 2001 and *Kim and Langley* [2001b] presented at the International Symposium on Kinematic Systems in Geodesy, Geomatics and Navigation, held in Banff, Alberta, Canada in June 2001.

REFERENCES

Baarda, W. (1968). *A Testing Procedure for Use in Geodetic Networks*, Publications on Geodesy, New Series, Vol. 2, No. 5, Netherlands Geodetic Commission.

Barnes, J. B., N. Ackroyd and P. A. Cross (1998). "Stochastic modeling for very high precision real-time kinematic GPS in an engineering environment." *Proceedings of the F.I.G. XXI International Congress*, Commission 6, Engineering Surveys, Brighton, U.K., 19-25 July, pp. 61-76.

Collins, J. P. and R. B. Langley (1999). "Possible weighting schemes for GPS carrier phase observations in the presence of multipath." Final contract report for the U.S. Army Corps of Engineers Topographic Engineering Center, No. DAAH04-96-C-0086 / TCN 98151, March, 33 pp.

Cox, D. T., K. W. Shallberg and A. Manz (1999). "Definition and analysis of WAAS receiver multipath error envelopes." *Navigation: Journal of the Institute of Navigation*, Vol. 46, No. 4, Winter, pp. 271-282.

Euler, H., and C. C. Goad (1991). "On optimal filtering of GPS dual-frequency observations without using orbit information." Bulletin Géodésique, Vol. 65, No. 2, pp. 130-143.

Han, S. (1997). "Quality-control issues relating to instantaneous ambiguity resolution for real-time GPS kinematic positioning." *Journal of Geodesy*, Vol. 71, No. 6, pp. 351-361.

Hartinger, H. and F. K. Brunner (1998). "Attainable accuracy of GPS measurements in engineering surveying." *Proceedings of the F.I.G. XXI International Congress*, Commission 6, Engineering Surveys, Brighton, U.K., 19-25 July, pp. 18-31.

Hofmann-Wellenhof, B., H. Lichtenegger, and J. Collins (1997). *GPS Theory and Practice*, 4th edition, Springer-Verlag.

Jin, X. X. (1996). "Theory of carrier adjusted DGPS positioning approach and some experimental results." Thesis. Delft University of Technology, Delft University Press, Delft, The Netherlands.

Kim, D. and R.B. Langley (2001a). "Estimation of the stochastic model for long-baseline kinematic GPS applications." *Proceedings of The Institute of Navigation 2001 National Technical Meeting*, Long Beach, CA, U.S.A., 22-24 January, pp 586-595.

Kim, D. and R.B. Langley (2001b). "Instantaneous real-time cycle-slip correction of dual-frequency GPS data", Proceedings of the International Symposium on Kinematic Systems in Geodesy, Geomatics and Navigation, Banff, Alberta, Canada, 5-8 June 2001, pp. 255-264.

Langley, R. B. (1997). "GPS observation noise." *GPS World*, Vol. 8, No. 6, April, pp. 40-45.

Leick, A. (1995). *GPS Satellite Surveying*, 2nd edition, John Wiley and Sons, Inc.

Rizos, C. (1999). *Quality Issues in Real-time GPS Positioning*, Final Report of the IAG SSG 1.154, July, (Available on-line at: <http://www.gmat.unsw.edu.au/ssg_RTQC/ssg_rtqc.pdf>).

Teunissen, P.J.G. and A. Kleusberg (editors) (1998). *GPS for Geodesy*, 2nd edition, Springer-Verlag.

Tiberius, C. C. J. M., N. Jonkman and F. Kenselaar (1999). "The stochastics of GPS observables." *GPS World*, Vol. 10, No. 2, February, pp. 49-54.

Tiberius, C. C. J. M. and F. Kenselaar (2000). "Estimation of the stochastic model for GPS code and phase observables." *Survey Review*, Vol. 35, No. 277, July, pp. 441-454.

Wang, J., M. P. Stewart and M. Tsakiri (1998). "Stochastic modeling for static GPS baseline data processing." *Journal of Surveying Engineering*, Vol. 124, No. 4, pp. 171-181.

Wang, J. (1999). "Stochastic modeling for real-time kinematic GPS/GLONASS positioning." *Navigation: Journal of the Institute of Navigation*, Vol. 46, No. 4, Winter, pp. 297-305.