An Assessment of Predicted and Measured Ionospheric Total Electron Content Using a Regional GPS Network

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BIOGRAPHIES

Attila Komjathy received his M.Sc.Eng. degree (1989) from the Department of Geodesy and Mining Surveying of the University of Miskolc, Hungary, and subsequently worked as a research assistant for the Hungarian Academy of Sciences. In 1991, he enrolled as a Ph.D. student in geodesy in the Department of Geodesy and Geomatics Engineering of the University of New Brunswick (UNB), Canada. Komjathy specializes in extraterrestrial positioning techniques for high precision geodesy and navigation. His current research focuses on the Global Positioning System (GPS) and its applications, concentrating on the study of the ionospheric effects on GPS signals using various modelling techniques.

Richard Langley is a professor in the Department of Geodesy and Geomatics Engineering at the UNB, where he has been teaching since 1981. He has a B.Sc. in applied physics from the University of Waterloo and a Ph.D. in experimental space science from York University, Toronto. After obtaining his Ph.D., Dr. Langley spent two years with the Department of Earth and Planetary Sciences of the Massachusetts Institute of Technology where he carried out research involving lunar laser ranging and very long baseline interferometry.

Dr. Langley has worked extensively with GPS. He is a co-author of the best-selling Guide to GPS Positioning published by Canadian GPS Associates and is a columnist for GPS World magazine. He has helped develop and present a number of seminar courses on GPS for both Canadian GPS Associates and the American-based Navtech Seminars Inc. Dr. Langley has consulted extensively in the field of GPS with private companies and government agencies both in Canada and abroad.

ABSTRACT

The signals from the satellites of the Navstar Global Positioning System (GPS) must travel through the earth's ionosphere on their way to GPS receivers on or near the earth's surface. To achieve the highest possible positioning accuracies from GPS, one must correct for the carrier phase advance and pseudorange group delay imposed on the signals by the ionosphere. Whereas these effects may be considered a nuisance by most GPS users, they will provide the ionospheric community with an opportunity to use GPS as a tool to better understand the plasma surrounding the earth.

The dispersive nature of the ionosphere makes it possible to measure its total electron content (TEC) using dual-frequency GPS observations collected by permanent networks of receivers. One such network is that of the International GPS Service for Geodynamics (IGS). We have used dual-frequency GPS pseudorange and carrier phase observations from six European stations in this network to derive regional TEC values.

In this research, we investigated the effect of using different elevation cutoff angles and ionospheric shell heights on TEC estimates and satellite-receiver instrumental biases. We found that using different elevation cutoff angles had an impact on TEC estimates at the 2 TEC unit (TECU) level. We also discovered that using different ionospheric shell heights has an effect on the ionospheric TEC estimates at about the 2 TECU level depending on geographic location and time of the day. We found no significant changes in the bias estimates using different elevation cutoff angles. We compared our TEC estimates with TEC predictions obtained by using the International Reference Ionosphere 1990 (IRI90) model. The results of this comparison are similar to those of other studies that were conducted using data sets at low solar activity times.

After processing the data from the 6 European stations collected over a 7 day period, we were able to follow highly varying ionospheric conditions associated with geomagnetic disturbances.

INTRODUCTION

One of the major error sources in GPS positioning is ionospheric refraction which causes signal propagation delays. The disturbing influences of the temporally and

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spatially varying ionization of the ionosphere have great impact on satellite geodesy, especially on GPS. Dual-frequency observations can be used to eliminate almost all of the ionosphere's effect. To correct data from a single-frequency GPS receiver for the ionospheric effect, it is possible to use empirical models. We are conducting an on-going study using such models.

After Newby [1992] investigated the International Reference Ionosphere (IRI86) model's performance, we decided to include the new IRI90 model [Bilitza, 1990] in our ionospheric research. We used Faraday rotation data as 'ground-truth' with which we compared the vertical ionospheric range error corrections predicted by the GPS navigation message [Klobuchar, 1986] and IRI90 models. Some of our results have been presented earlier [Komjathy et al., 1995a]. Based on the comparison between the Broadcast and IRI90 models, we concluded that both for day-time and night-time periods the IRI90 model appeared to be more accurate than the Broadcast model. This conclusion is specific to low solar activity and mid-latitude conditions based on a limited set of data [Komjathy et al., 1995b].

Since the availability of Faraday rotation data for use as 'ground-truth' is limited, we decided to use dual-frequency pseudorange and carrier phase GPS measurements to infer ionospheric TEC.

The literature of relevance to this research is large. Early studies used single station observations to estimate the line-of-sight pseudo-TEC which is the sum of the satellite-receiver instrumental biases and the actual line-of-sight TEC (e.g., Lanyi and Roth [1988], Coco et al. [1991]). The necessity to produce global ionospheric maps with more accurate TEC and bias estimates has led the ionospheric community to use multi-site fitting techniques. Several research groups have started producing regional or global scale TEC maps along with satellite-receiver instrumental biases depending on the type of "ionospheric" observable used. For generating "ionospheric" observables, one can use undifferenced dual-frequency pseudorange, undifferenced dual-frequency carrier phase observations or these two combined. Most research groups use the combined (phase-levelling) technique in which case the integer ambiguity afflicted differences of the L1 and L2 (L1-L2) carrier phase measurements are adjusted by a constant value determined for each phase-connected arc of data using precise pseudorange measurements. The L1-L2 "ionospheric" observable has a noise level 1-2 orders of magnitude below the pseudorange "ionospheric" observable. The technique has been described by Wilson and Mannucci [1994], Runge et al. [1995] and others. It is widely used to estimate various ionospheric model parameters as well as satellite-receiver instrumental biases (see, e.g., Gao et al. [1994] and Sardon et al. [1994]). It is also feasible to use double-differenced L1-L2 carrier-phase observations to estimate global or regional ionospheric models [Schaer et al. 1995]. The advantage of this technique is that by using the double-differenced "ionospheric" observable, one does not have to estimate the satellite-receiver instrumental biases as they are differenced away. The price we have to pay is that we lose some of the resolution of the ionospheric signal. We have therefore chosen to use the former technique.

ESTIMATION STRATEGY

The ionospheric measurements from a GPS receiver can be modelled with the commonly used single-layer ionospheric model using the observation equation (1):

$$I_r^s(t_k) = M(e_r^s)\left[a_{0,r}(t_k) + a_{1,r}(t_k)d\lambda_r^s + a_{2,r}(t_k)d\varphi_r^s\right] + b_r + b^s \tag{1}$$

where

$I_r^s(t_k)$ is the L1-L2 phase measurement at epoch t_k made by receiver *r* observing satellite *s*,

$M(e_r^s)$ is the thin-shell elevation mapping function projecting the line-of-sight measurement to the vertical (see, e.g., Schaer et al. [1995]),

$a_{0,r}, a_{1,r}, a_{2,r}$ are the parameters for spatial linear approximation of TEC to be estimated assuming a first-order Gauss-Markov stochastic process [Gail et al. 1993],

$d\lambda_r^s = \lambda_r^s - \lambda_0$ is the difference between a subionospheric point (the intersection of the ray path of a signal propagating from the satellite to the receiver with a thin spherical shell) and the mean longitude of the sun,

$d\varphi_r^s = \varphi_r^s - \varphi_r$ is the difference between the geomagnetic latitude of the subionospheric point and the geomagnetic latitude of the station, and

b_r, b^s refer to the receiver and satellite instrumental biases respectively.

The parameters $a_{0,r}, a_{1,r}, a_{2,r}$ in equation (1) are estimated using a Kalman filter approach. The prediction and update equations for the state estimation are described by Schwarz [1987], Coster et al. [1992] and van der Wal [1995]. We allowed the model to follow a relatively high 1 TECU per 2 minutes change in the total electron content which resulted in the process noise variance rate of change to be $0.008 \cdot \mathrm{TECU}^2$ / second characterizing the uncertainties of the dynamic ionospheric model. For the variance of the measurement noise, we used 1 $TECU^2$ which describes the uncertainty in the observations.

In our investigation, we did not use receivers with calibrated instrumental delays. We estimated the combined satellite-receiver instrumental delays for one station of the network. The ionospheric research community tend to use station Madrid for that purpose so that it is easier to exchange results among each other. In our network solution, we need to estimate additional biases for the other stations based on the fact that the other receivers have different instrumental delays. Therefore, for each station other than the station Madrid an additional bias parameter was estimated which is the difference between the receiver instrumental delays between a station in the network and station Madrid. This technique is described by Sardon et al. [1994].

Since the reason for the ionosphere's existence is the interaction of ionizing radiation (principally from solar ultraviolet and x-ray emissions) [Langley, 1992] with the earth's atmosphere and magnetic field, we chose a solar-geomagnetic reference frame based on sun-fixed longitude and geomagnetic latitude. Mannucci et al. [1995] concluded that the ionosphere varied much more slowly in a sun-fixed reference frame compared to an earth-fixed one and resulted in more accurate ionospheric delay estimates when using Kalman-filter updating.

THE DATA SET

With this method, we analysed dual-frequency GPS data sets from the European region consisting of 6 stations of the International GPS Service for Geodynamics (IGS) network collected by Turbo Rogue receivers. The stations are listed in Table 1 and are identified on the map in Figure 1.

Station	Geographic lat. in deg.	Geographic long. in deg	Geomagnetic lat. in deg
Madrid, Spain	40.4	-4.2	42.8
Grasse, France	43.7	6.9	45.4
Matera, Italy	40.6	16.7	40.5
Brussels, Belgium	50.8	4.4	52.7
Wettzell, Germany	49.1	12.9	49.4
Onsala, Sweden	57.4	11.9	57.4

Table 1. List of IGS stations used for data analysis.

The differences in geomagnetic latitudes of stations Madrid, Grasse, and Matera are less then 5 degrees, and 3.3 degrees in case of stations Brussels and Wettzell. Therefore, we can identify three distinct latitude regions in our test network (1. Madrid, Grasse, Matera; 2. Brussels, Wettzell; 3.Onsala).

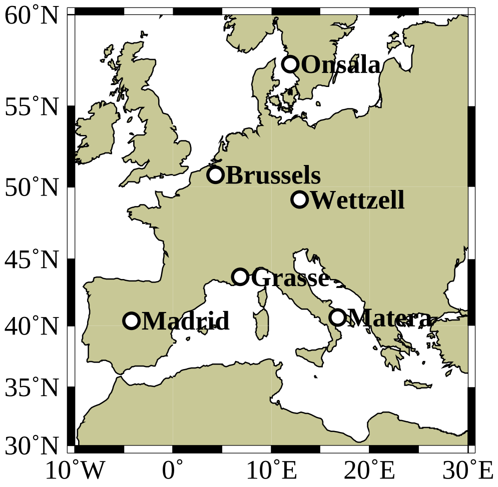


Figure 1. Locations of IGS stations used for data analysis.

We processed 7 days' worth of data from all 6 stations spanning the time period 15 to 21 October 1995 (GPS week 823) during which a geomagnetic disturbance occurred [NGDC, 1995]. The planetary equivalent amplitude of magnetic activity a_p suggests that the magnetic disturbance started on 18 October, 1995 and lasted for about 6 days until 23 October, 1995. The peak (a_p = 111) occurred on 19 October, 1995. We chose to process data sets for the whole of GPS week 823 encompassing both magnetically quiet and disturbed conditions.

RESULTS AND DISCUSSION

We used PhasEdit version 2.0 automatic data editing program to detect bad points and cycle slips, repair cycle slips and adjust phase ambiguities using the undifferenced data. The program takes advantage of the high precision dual-frequency pseudorange measurements to adjust L1 and L2 phases by an integer number of cycles to agree with the pseudorange measurements [Freymueller, 1995].

The University of New Brunswick's DIfferential POsitioning Program (DIPOP) package was extensively modified to estimate ionospheric parameters, and satellite-receiver instrumental biases using a Kalman filter algorithm.

We investigated the effect of using different elevation cutoff angles on the TEC estimates and satellite-receiver instrumental biases. Since the elevation dependence allows us to separate TEC from satellite-receiver instrumental biases, it is crucial to know what is the minimum elevation cutoff angle we can use without mismodelling the TEC due to the higher noise on observations with lower elevation angles. Or alternatively, what is the maximum elevation cutoff angle that is still acceptable without mismodelling the satellite-receiver instrumental biases. We considered three different elevation angle cutoffs: 15, 20, and 25 degrees. Figure 2 shows the differences in TEC estimates of 15 and 20 degrees with respect to those computed using a 25 degree cutoff angle for Madrid, Brussels, and Onsala. These differences are displayed in separate graphs to illustrate the small differences. We used 25 degree elevation cutoff angle as a reference for the other two since this solution was expected to be the least noisy as we can see by looking at the r.m.s. of L1-L2 “ionospheric” residuals in Table 2. These stations represent the three different geomagnetic latitude regions from the network of stations we investigated (see Table 1).

It appears that the peak-to-peak variation of the differences formed by the 15 and 25 degree elevation cutoff angle solutions is larger than the one formed by the 20 and 25 degree solutions which is what we would expect knowing that there are noisier observations included when computing the solution with a 15 degree cutoff angle. Using different elevation cutoff angles produces maximum differences in TEC estimates that can be characterized by a bias of 0.3 TECU and standard deviation of 0.5 TECU. This maximum bias and associated standard deviation occured at station Onsala.

Another parameter that affects the TEC estimation is the height of ionospheric shell which plays an important role in computing the coordinates of the subionospheric points. It is also an input parameter of the $M(e_r^s)$ mapping function - (see equation (1)). The single-layer ionospheric model assumes that the vertical TEC can be approximated by a thin spherical shell which is located at a specified height above the surface of earth. This altitude is often assumed to correspond to the maximum electron density of the ionosphere. Furthermore, it is usually assumed that the ionospheric shell height has no temporal or geographical variation and therefore it is set to a constant value regardless of the time or location of interest. In our investigation we looked at fixed heights of 300, 350, and 400 km and also included variable heights computed by the IRI90 model using F2 layer peak heights.

The TEC differences shown in Figure 2 were obtained using a 350 km shell height. In Figure 3, we have plotted the differences between corresponding TEC estimates for the three representative stations using different ionospheric shell heights. We used a 20 degree elevation cutoff angle to generate the TEC estimates used in this figure. The differences tend to decrease as further north the station is. Since the IRI90’s F2 layer critical heights take the geographical and temporal variation into account, there is a larger periodic variation of the differences involving IRI90 heights. By looking at Figure 3, the largest differences between two different solutions appear to be at the 2 TECU level at the Madrid station.

We computed the mean of the daily satellite-receiver instrumental biases for all 7 days. We also obtained a set of bias estimates computed by Deutsche Forschungsanstalt für Luft und Raumfahrt (DLR) Fernerkundungsstation, Neustrelitz, Germany [DLR, 1995]. After computing the mean of the corresponding values obtained from DLR for all 7 days we computed the differences of the corresponding biases. After that, we calculated the mean and standard deviation of the differences computed by UNB’s (using different elevation cutoff angles and different ionospheric shell heights) and DLR’s results (using 20 degrees elevation cutoff angle with 355 km ionospheric shell height [Sardon et al., 1994]). The means and standard deviations are listed in Table 2. The differences between UNB’s and DLR’s individual instrumental biases range between 0.01 and 1.10 ns. It is clear that the mean of the differences in biases decreases as higher ionospheric shell heights are chosen. It appears that choosing different elevation cutoff angles has no significant impact on the bias differences. Largest bias differences occur using the F2 layer critical heights computed by the IRI90 model. This is mainly because IRI90 predicts the critical heights to be between 220 and 340 km depending on location and time of the day. The differences between the two institutions’ results may be due to the fact that the UNB network consists of only 6 stations. A larger number of stations would provide more accurate instrumental bias values as well as TEC estimates. Our software has been designed to easily accommodate more stations in future data analyses.

The magnetic disturbance on day 292 affected the diurnal variation of the total electron content. This can be seen in Figure 4. As an example, we display the diurnal variation of our estimates of TEC for two days at station Madrid: day 288 when the magnetic field was quiet and day 292 when the magnetic disturbance reached its peak.

	Elevation angle cutoff in degrees								
shell height in km	15			20			25		
	mean of diff. in ns	s.d. of diff. in ns	r.m.s. of UNB res. in TECU	mean of diff. in ns	s.d. of diff. in ns	r.m.s. of UNB res. in TECU	mean of diff. in ns	s.d. of diff. in ns	r.m.s. of UNB res. in TECU
IRI	0.66	0.38	1.09	0.67	0.40	1.00	0.66	0.40	0.91
300	0.54	0.34	1.07	0.58	0.37	0.98	0.59	0.38	0.90
350	0.44	0.31	1.06	0.50	0.34	0.97	0.53	0.36	0.90
400	0.35	0.28	1.04	0.43	0.32	0.96	0.47	0.34	0.89

Table 2. Summary of comparison of instrumental bias differences between UNB and DLR and r.m.s. of UNB ionospheric residuals.

We were interested in finding out whether our algorithm was able to follow the rapidly changing ionospheric conditions on day 292. The two upper panels of Figure 4 displays three curves representing coefficients $a_{0,r}, a_{1,r}, a_{2,r}$ (see equation (1)) with the corresponding error bars. These coefficients represent a constant offset, local time slope and latitude slope of the modelled vertical TEC, respectively, in the vicinity of a station. The two upper panels also show vertical TEC at station Madrid has been computed by evaluating the vertical TEC model in equation (1) (see expression in brackets). The shape of the diurnal curves on day 288 and 292 are significantly different. On day 288, the largest TEC value is around 15 TECU whereas on day 292, the largest TEC is around 25 TECU. On the bottom panel of Figure 4, we plotted the UNB "ionospheric" residuals for all 6 stations for day 292.

In Figure 5, we display all TEC diurnal curves for all stations for all 7 days. We have also plotted the TEC values predicted by the IRI90 model. It is very interesting to see that on day 292, at stations Madrid, Grasse and Matera, the peak of TEC values increased considerably compared to peaks for the previous days. On the other hand, for stations Brussels, Wettzell and Onsala, the GPS-derived TEC estimates show peaks with smaller size than the ones on the previous days. On the bottom panel of Figure 5, we indicate the planetary equivalent amplitude of magnetic field variation [NGDC, 1995]. We can see that large TEC variations on day 292 were preceded by the magnetic field disturbance starting on day 291. Day 292 seemed to be the most variable among the 7 days under investigation. The bottom panel of Figure 4 suggests that there are no major difficulties for our algorithm to follow ionospheric variations induced by the geomagnetic field disturbance. This statement may only hold for the current low solar activity times. The effect of major magnetic field variations on GPS-derived TEC estimates is yet to be investigated.

In Figure 5, we have also displayed the total electron content computed by the IRI90 model (darker, smoother curve).

	Mean of differences in TECU	S.d. of differences in TECU
Madrid	-0.7	1.5
Grasse	-0.4	1.8
Matera	0.5	2.3
Brussels	0.2	1.5
Wettzell	1.1	1.7
Onsala	0.3	1.5

Table 3. Comparison of GPS-derived TEC estimates with IRI90 predictions.

We computed the differences of IRI90 predicted and GPS-estimated TEC values. The means of the differences and the standard deviations for the 6 stations are summarized in Table 3. When computing the mean of differences, we used all 7 days of data. The values seem to be overly optimistic compared to the results described by Komjathy [1994b]. This may be due to the fact that our current investigation represents lower solar activity conditions compared to those of the data in Komjathy [1994b] which represented solar activity conditions from April 1994 to January 1995. This is also supported by the research conducted by Newby [1992]. In his thesis, Newby looked at high, medium and low solar activity conditions. Newby showed that the IRI86 model performance based on low solar activity conditions showed good agreement with Faraday rotation data at the 1.8 TECU level.

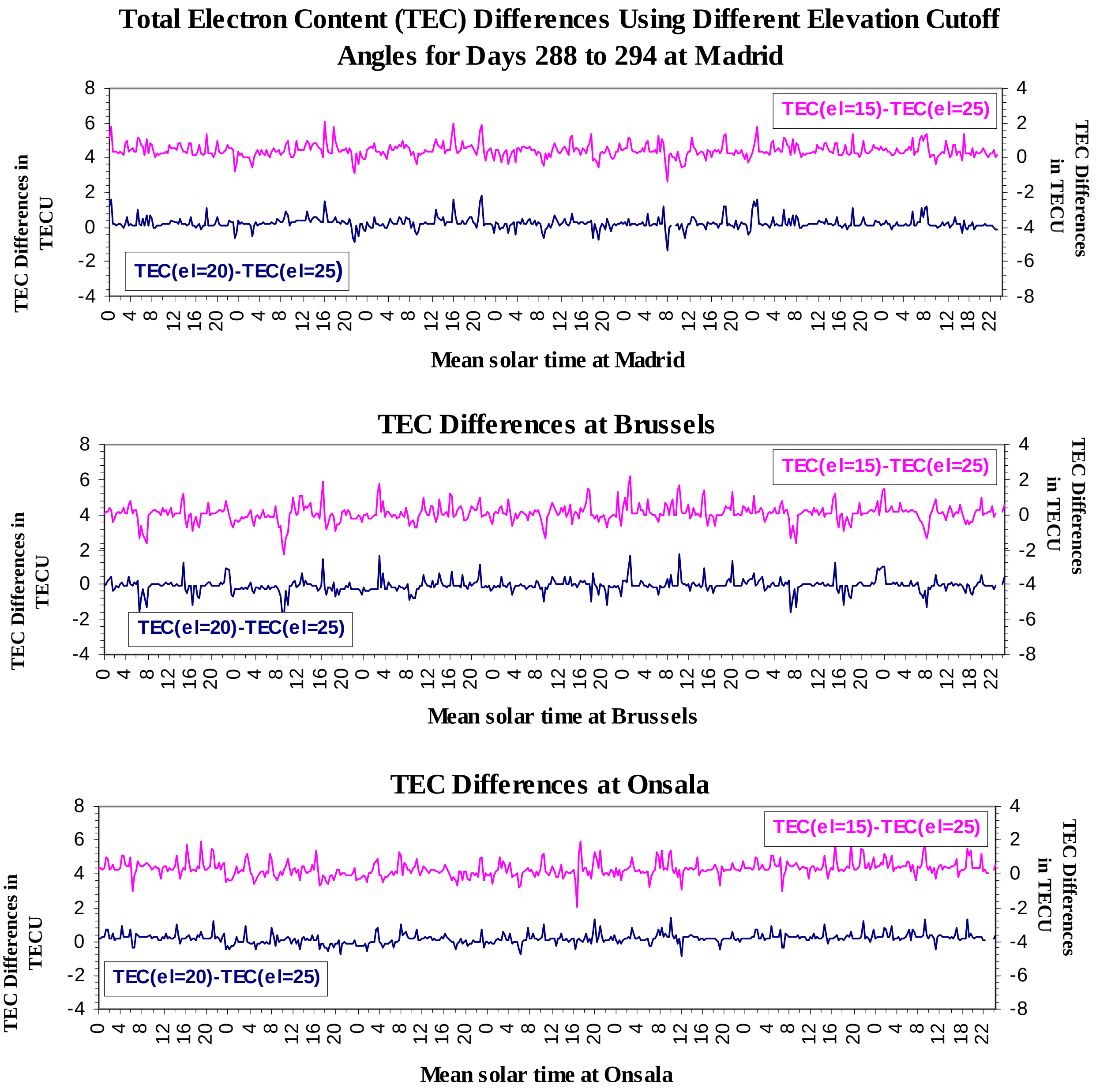


Figure 2. Comparison of TEC estimates using different elevation cutoff angles.

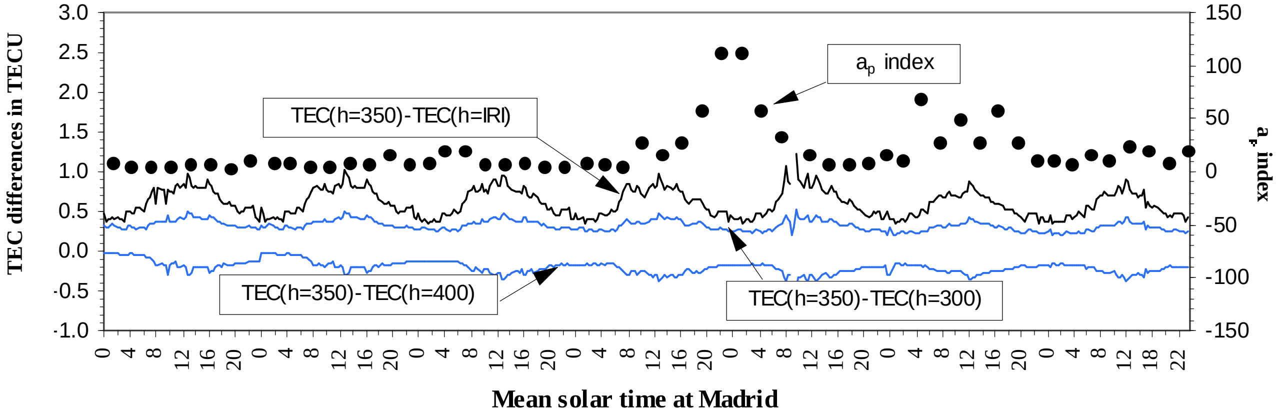
CONCLUSIONS

We investigated the effect of using different elevation cutoff angles and ionospheric shell heights on TEC estimates and satellite-receiver instrumental biases. We found that using different elevation cutoff angles had an impact on TEC estimates at the 2 TECU level. Also, several ionospheric shell heights were looked at. We found that at the 2 TECU level, the ionospheric estimates using different heights agree depending on geographic location and time of the day. We also compared our bias estimates with results from DLR. We found an agreement at the 0.5 ns level. The differences of the biases compared by UNB and DLR indicate that using different elevation cutoff angles appears not to have a significant influence on the biases. However, we found that using higher ionospheric shell heights decreased the bias differences significantly. We also compared our TEC estimates with TEC predictions obtained by using the IRI90 model.

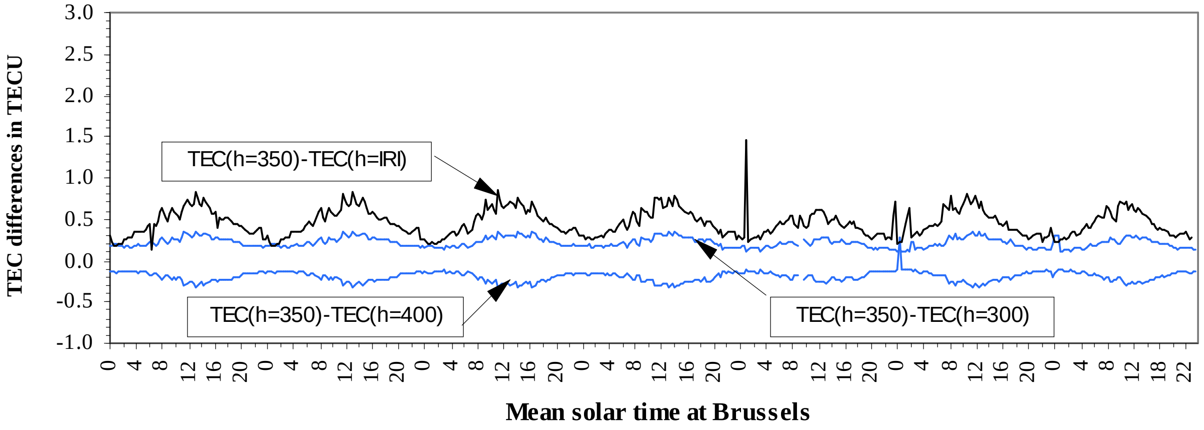
We managed to follow highly varying ionospheric conditions due to a magnetic disturbance. Due to the limited data set investigated, it is also important to point out that the results presented here are only specific for mid-latitude stations at low solar activity conditions.

In the future, we hope to be able to provide TEC updates for the IRI90 model by using dual-frequency GPS measurements after first establishing dependencies of TEC on latitude, longitude and local time. We will also look at the effect of seasonal variations as well as magnetic and solar activity on these differences.

Total Electron Content Differences Using Different Shell Heights (h) for Days 288 to 294 at Madrid



TEC Differences at Brussels



TEC Differences at Onsala

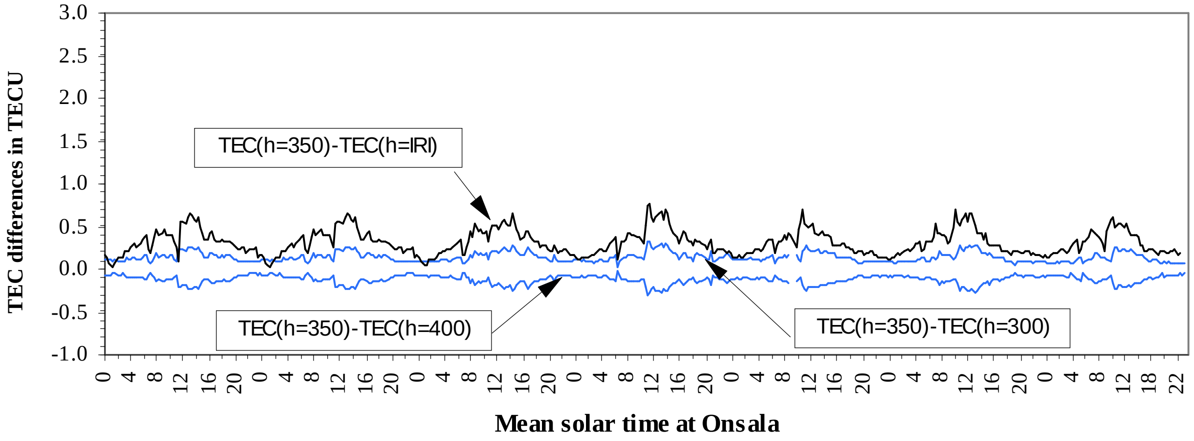


Figure 3. Comparison of TEC estimates using different ionospheric shell heights.

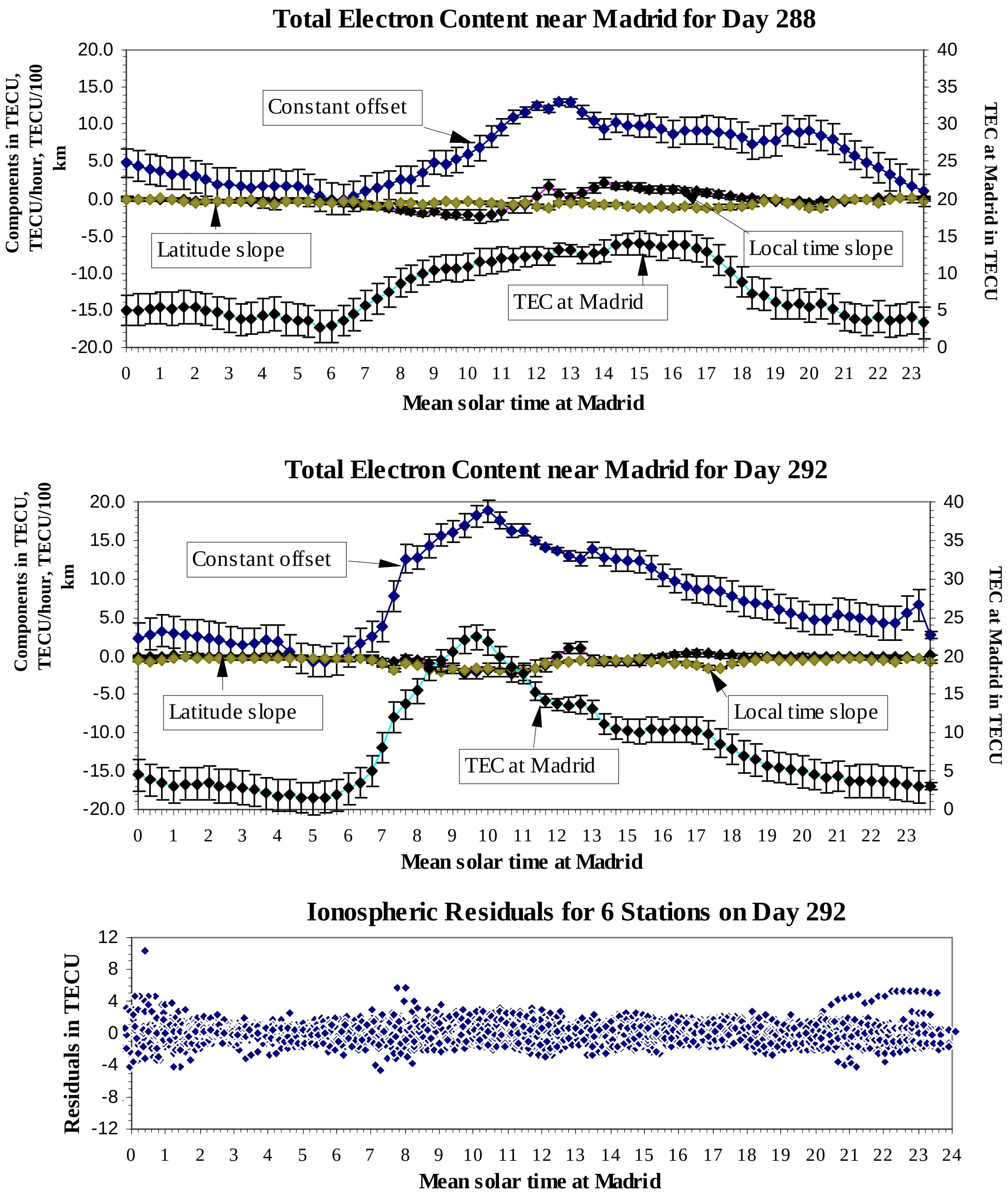


Figure 4. An example of the diurnal variation of TEC for magnetically quiet and disturbed days, and ionospheric residuals.

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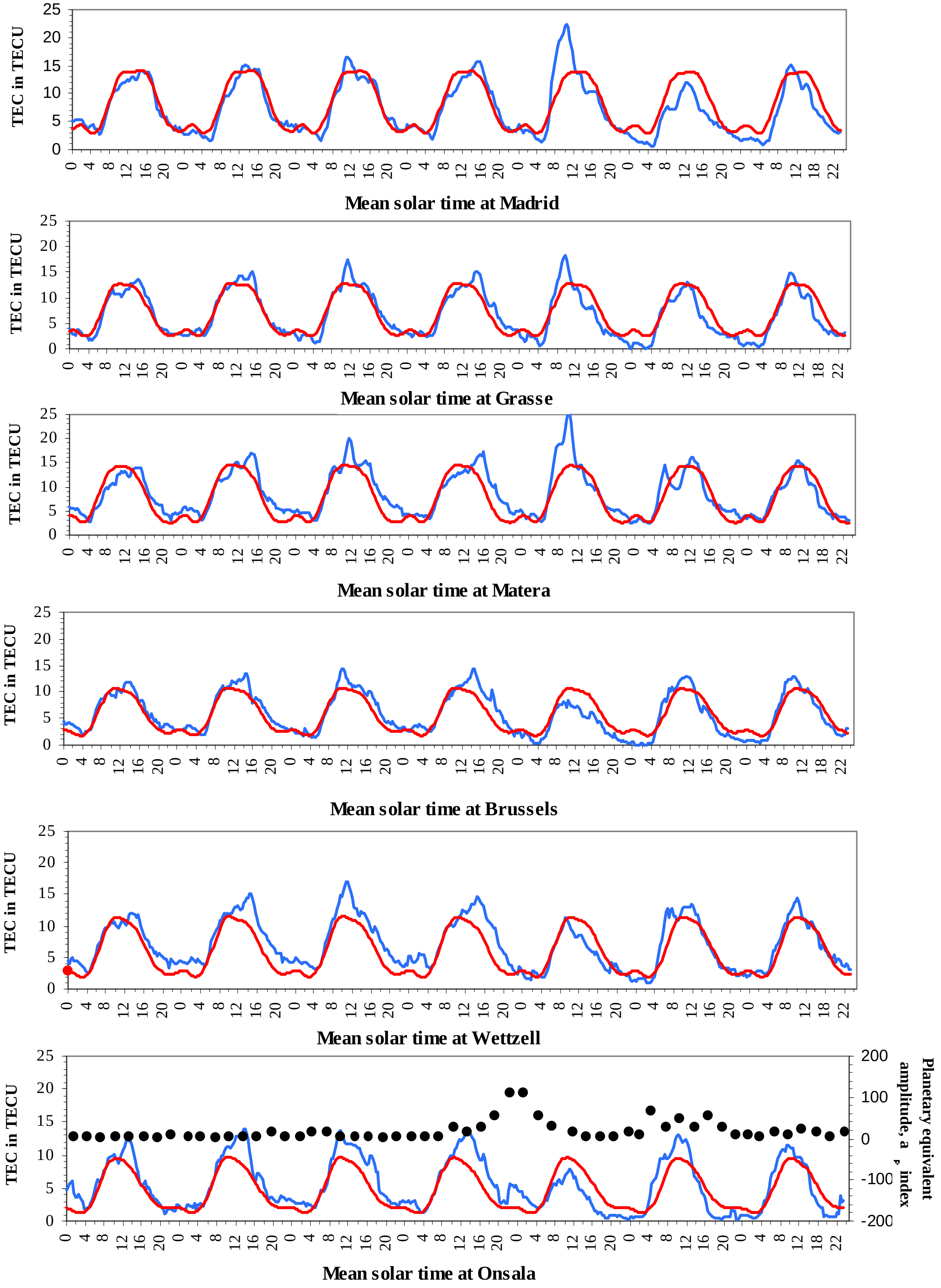

Figure 5. Diurnal variation of TEC at 6 stations for 7 consecutive days using a network of IGS stations (darker, smoother curve represents the IRI90 predictions)

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