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The Orbits of GPS Satellites

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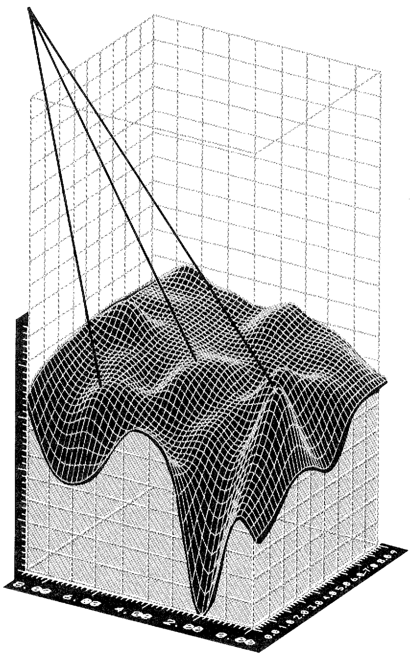

"Innovation" is a regular column in GPS World *featuring discussions on recent advances in GPS technology and its applications as well as on the fundamentals of GPS positioning. This time we look at the orbits of GPS satellites.*

This column is coordinated by Richard Langley and Alfred Kleusberg of the Department of Surveying Engineering at the University of New Brunswick. As always, we welcome your comments and suggestions of topics for future columns.

A Navstar Global Positioning System receiver makes measurements of the distance between its antenna and a number of GPS satellites. By combining those measurements with knowledge of the positions of the satellites, the receiver is able to determine its own position. Like the moon about the earth or the planets about the sun, man-made satellites travel in orbits that are shaped by the inexorable pull of gravity. In order to accurately calculate the position of a satellite in its orbit at any time, gravity and other smaller forces acting on the satellite must be carefully taken into account. In this column we'll describe the forces that determine a satellite's orbit, see how orbits are characterized, and look at how orbital information is conveyed to the GPS receiver. But in order to understand the orbits of satellites a little better, we'll first review a bit of the physics that govern their behavior.

KEPLER'S LAWS

On the 30th of September, 1990, there were 6,681 detectable objects in orbit about the earth, as reported by the North American Aerospace Defense Command. Included were 14 active Navstar GPS satellites, a number that since has grown to 16. The orbits of the GPS satellites, along with those of the several hundred other functioning satellites and the thousands of pieces of space debris, are governed by the three laws of orbital motion discovered by the German astronomer Johannes Kepler in the seventeenth century.

Kepler believed in a Copernican universe, one in which the sun rather than the earth was at the center. But he was not content to be just a believer; he wanted to put the Copernican model to the test. For such a test he needed a set of accurate observations of the positions of the planets. Such a set was being accumulated by the great observational astronomer, Tycho Brahe, first in his native Denmark and subsequently in Prague. Kepler became Tycho's assistant and, on Tycho's death, acquired (despite a court challenge by Tycho's widow) the books containing all of Tycho's observational data.

Kepler attempted to fit the Copernican model to the data. He started work first on the motion of Mars because the data on Mars were so extensive and because its complex motion, as observed from the earth, would be a good test for any model. But try as he might, Kepler was left with a discrepancy of eight seconds of arc in the position of Mars when he tried to fit Mars's orbit using the "circles upon circles" of the Copernican model (in this theoretical model, the planets were supposed to travel in small circular orbits about central points which in turn orbited about the sun). It took Kepler eight years of wrestling with Tycho's data before he discovered that the orbit of Mars was actually an ellipse with the sun at one of the foci. This discovery has become known as Kepler's first law of planetary motion.

In the course of his analyses, Kepler discovered another important law governing the motion of the planets: a line joining the sun to a planet sweeps out equally sized areas in a given interval of time regardless of where the planet is in its orbit. This is Kepler's second law. Both of these laws, specifically referring to Mars, were published by Kepler in 1609 in his book *Astronomia Nova,* or *The New Astronomy*.

In 1618, Kepler announced that these two laws also governed the motion of the other planets, the earth's moon, and the four newly discovered moons of Jupiter. One year later in a book entitled *Harmonices Mundi,* or *The Harmony of the World,* Kepler triumphantly published his third law: the square of the time taken by a planet to complete its orbit is proportional to the cube of its mean distance from the sun.

Although Kepler obtained his laws with a certain amount of numerology and without a workable theory for their explanation, Sir Isaac Newton, only half a century later, showed that Keplerian laws could be put on a firm scientific foundation and that they were in fact derivable from his theory of universal gravitation. Newton showed that Kepler's third law actually related the period of a planet's elliptical orbit to the size of the ellipse and showed how the proportionality factor was related to the product of the sum of the masses of the sun and the planet and the gravitational constant, one of the immutable constants of nature. Newton also hypothesized that, given the correct initial velocity, a projectile fired from the earth would go into orbit about it. Newton actually foretold of artificial satellites almost two hundred years before the first one was launched.

THE KEPLERIAN ELEMENTS

Kepler's laws tell us that the path of a satellite orbiting about the earth is an ellipse with one of its foci coincident with the earth's center of mass. Given the position and velocity of a satellite at some arbitrary time, we can use the laws to predict a future position of the satellite. However, an alternative and often more useful way of representing the orbit of a satellite is to use the six so-called Keplerian elements.

Two of these elements, a, the *semimajor axis*, and e, the *eccentricity,* give the size and shape of the orbital ellipse. The semimajor axis is one-half of the longest dimension of the ellipse. The eccentricity is a measure of its "ovalness" and is a dimensionless number between 0 and 1. A circle has an eccentricity of 0.

The orientation of the orbit in space with respect to the "fixed" stars is given by three more parameters: i, the *inclination*, which is the angle between the orbit plane and the earth's equator (the vertex of this angle and the ones defined below coincides with the center of the earth); Ω, the *right ascension of the*

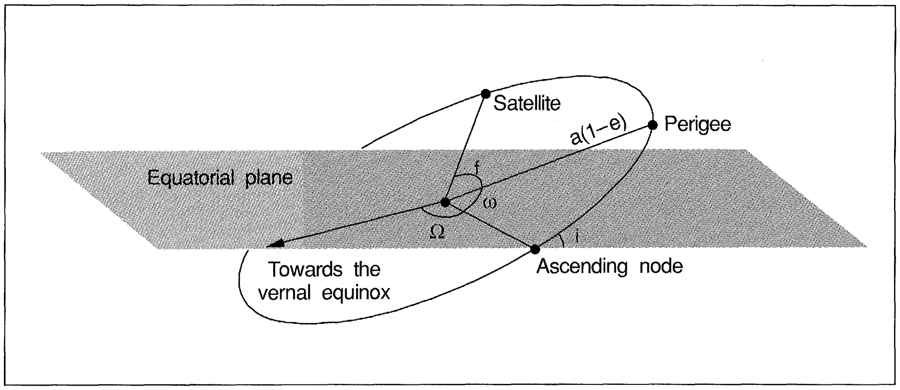

Figure 1. The elements of a Keplerian orbit

ascending node, which is the angle measured in the earth's equatorial plane between a reference direction in space called the first point of Aries or the vernal equinox (which coincides with the position of the sun the instant spring begins in the Northern Hemisphere) and the *ascending node*, the point on the satellite's orbit where the satellite crosses the plane of the equator, moving from below the equator to above it; and ω, the *argument of perigee*, which is the angle measured in the plane of the orbit between the ascending node and the perigee, the point on the orbit that is closest to the center of the earth. A little bit of geometry shows us that the distance from the earth's center to the perigee is given by $a(1-e)$, as illustrated in Figure 1.

The sixth Keplerian element tells us where on the orbital ellipse the satellite is at a certain time. There are several different ways of specifying this element. The most fundamental way is to specify the *true anomaly* of the satellite at a reference time called the epoch. The true anomaly is the angle between the perigee and the satellite. Given the true anomaly at some time and the semimajor axis and the eccentricity of the orbital ellipse, we can compute the distance of the satellite from the focus — the orbit radius. The sum of the true anomaly and the argument of perigee gives the position on the orbit with respect to the ascending node. This quantity is called the *argument of latitude* and is used, for example, in the computations that a GPS receiver performs to establish the position of a satellite.

Alternatively, the *mean anomaly* may be specified. This is the angle between the perigee and a fictitious satellite with the same orbital period as the real satellite but moving with a constant speed. This constant speed is called the *mean motion* of the satellite. Given one kind of anomaly, the other can be readily computed. (The use of the word *anomaly* to describe these angles in orbital mechanics came about because of the expectation of early astronomers that all orbits would be circular.)

ORBIT PERTURBATIONS

We know now that Kepler's laws fully describe the motion of a satellite only if the satellite is in orbit about a body that has a "perfect" — or spherically symmetric — gravitational field, called a central field. The earth's gravitational field is noncentral and, in fact, is quite complex. This complexity is dominated by the earth's oblateness — the equatorial radius of the earth is some 20 kilometers larger than its polar radius. This equatorial bulge, together with the higher order components of the earth's gravitational field, perturbs the orbits of satellites. This means that the Keplerian elements, in general, will change with time.

Other forces also act on a satellite to change its orbit. Low orbiting satellites are significantly affected by the earth's atmosphere. Although quite tenuous at a height of several hundred kilometers, the atmosphere acts to decelerate a satellite. Deceleration results in a gradual decay of the orbit until, eventually, the satellite "de-orbits" and burns up in the lower atmosphere. This happens when the satellite reaches an altitude of about 120 kilometers, which corresponds to an orbital period of about 87 minutes. Although all satellites experience the atmosphere's retarding force, or drag, its effect on satellites in high orbits, such as the GPS constellation, is negligible.

Another perturbation affecting satellite orbits is solar radiation pressure. The photons that make up sunlight exert a minute force when they collide with a satellite. A photon is either absorbed or reflected in the collision and transfers some of its momentum to the satellite. The effect of this force depends on the amount of incident radiation and the mass and shape of the satellite. The effect on lighter and larger satellites is greater than that on heavier and smaller ones. Although the force is small compared to that of the central gravitational field, its effect accumulates over time and gradually changes the satellite's orbit.

Some other forces affecting satellite orbits include the gravitational fields of other bodies such as the sun and the moon, the earth's magnetic field, and the small changes to the earth's gravitational field caused by the tides.

Of course, the orbits of satellites can also be deliberately changed through controlled firings of onboard rocket engines. Large engines are used to inject the satellite into the desired orbit. Smaller engines, called reaction control thrusters, are used to make small adjustments to an orbit. Thrusters may be used to counteract the effects of perturbing forces and thereby maintain a satellite in a particular orbit.

Newton hypothesized that, given the right initial velocity, a projectile fired from the earth would go into orbit around it.

A maneuver involving a thruster firing is called a *delta-V* because it results in a change of the satellite's velocity. The engines on board a number of the GPS satellites were recently used to change the relative positions of the satellites within their orbit planes, a procedure known as *rephasing*. The rephased positions give better satellite coverage during the buildup to the fully operational constellation.

Table 1. Ephemeris parameters in the navigation message

$\sqrt{a}$	Square root of the semimajor axis
e	Eccentricity
i_0	Inclination angle at the reference time
Ω_0	Longitude of the ascending node at the beginning of the GPS week
ω	Argument of perigee
M_0	Mean anomaly at the reference time
Δn	Correction to the mean motion computed using $\sqrt{a}$
i-dot	The rate of change of the inclination with time
Ω-dot	The rate of change of the right ascension of the ascending node with time
C_{uc}, C_{us}	Amplitude of correction terms for the computed argument of latitude
C_{rc}, C_{rs}	Amplitude of correction terms for the computed orbit radius
C_{ic}, C_{is}	Amplitude of correction terms for the computed inclination angle
t_{oe}	Ephemeris reference time

Each GPS satellite is oriented in space so that its transmitting antenna always points toward the earth. This requires the satellite to rotate once per orbit about an axis perpendicular to the orbit plane. However, disturbing torques caused by some of the perturbing forces mentioned earlier can result in additional motions about the satellite's center of mass that could lead to the antennas pointing away from the earth.

To control its orientation, or attitude, each satellite has four reaction wheels, commonly known as flywheels. These flywheels are used in conjunction with the thrusters and with a set of magnetic coils that interact with the earth's magnetic field. Compensating torques are generated by accelerating the flywheels. When the flywheels reach their maximum speed, they must be unloaded by firing the thrusters or activating the magnetic coils in a procedure known as a *momentum dump*.

LAUNCHING GPS SATELLITES

The prototype, or Block I, GPS satellites were launched from Vandenburg Air Force Base in California using Atlas F rockets. The 10 satellites were placed in nominally circular orbits with semimajor axes of about 26,560 kilometers. The satellites were positioned in two orbit planes with inclinations of about 64° to provide maximum coverage for the main military testing area for GPS, the Yuma Proving Grounds in Arizona.

Although the Block I satellites were launched using an expendable launch vehicle, the system operators originally intended to launch the Block II satellites, up to three at a time, using the Space Transportation System — the Space Shuttle. But after the Challenger accident, the decision was made to use an expendable launch vehicle and a new rocket, the Delta II, was developed for this purpose. Perched atop Delta IIs, the Block II satellites are launched from the Cape Canaveral Air Force Station next door to the Kennedy Space Center.

The satellites achieve their final orbits in steps. The first and second stages of the rocket together with the nine solid-fuel strap-on booster engines put the third stage of the rocket and the attached satellite into an elliptical orbit with a perigee height of about 180 kilometers and an apogee height of about 870 kilometers. (The *apogee* is the point on the orbit furthest from the earth.) The rocket's third stage, called a Payload Assist Module (PAM), is then used to increase the apogee height of the orbit so that it matches the height of the desired final orbit, about 20,200 kilometers, above the earth's surface.

At this point, the satellite is in a highly elliptical orbit, called a transfer orbit. The PAM is jettisoned and the satellite's orbit insertion engine is fired to put the satellite into an approximate final orbit. Small adjustments are then made using the satellite's thrusters. As described in the "Innovation" column in the November/December 1990 issue of *GPS World*, this orbit is nominally circular (maximum eccentricities are about 0.01 with a semimajor axis of about 26,560 kilometers and an inclination of about 55°). The resulting orbital periods of the satellites are within a minute or so of exactly one-half of a sidereal day (which is approximately four minutes shorter than a solar day). In order to achieve global, 24-hour-a-day coverage, four satellites will be placed in each of six orbital planes, named A through F. The right ascensions of the ascending nodes of orbits in adjacent planes are separated by 60°. This arrangement of the satellite constellation results in at least six satellites being visible from any point on the earth at all times.

ORBIT DATA

In order for a GPS receiver to compute its position in real time, it must know the positions of the satellites. These positions are established by the GPS Operational Control System (OCS) and provided in the navigation messages broadcast by the satellites. The OCS, which is operated by the Air Force Space Command, includes five tracking stations, spaced in longitude around the globe. Three of these stations are on small islands primarily used for military purposes: Ascension Island, Diego Garcia, and Kwajalein. The other two stations are in Hawaii and at Colorado Springs, Colorado. The Colorado Springs site also acts as the Master Control Station (MCS). (A back-up MCS is located at the Onizuka Air Force Base in Sunnyvale, California.)

The MCS collects the pseudorange and carrier-phase data obtained by the tracking stations and, with sophisticated software models, predicts the future orbits of the satellites. A computed orbit, called an *ephemeris*, is uploaded to the corresponding satellite using facilities at the Ascension Island, Diego Garcia, or Kwajalein tracking stations and is subsequently broadcast by the satellite.

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The OCS computes its orbits by fitting

Keplerian elements to the tracking data together with some additional parameters to account for the perturbations of the orbits. The 16 parameters of the broadcast ephemeris are listed in Table 1 and illustrated in Figure 2. The square root of the semimajor axis is used rather than the semimajor axis itself to speed up the GPS receiver's calculation of the satellite's position. Likewise, the longitude of the ascending node rather than its right ascension is used in the message. The three parameters Δn, i-dot, and Ω-dot account for the linear changes in the orbit with time; the six C-values are the amplitudes of sinusoidally varying correction terms.

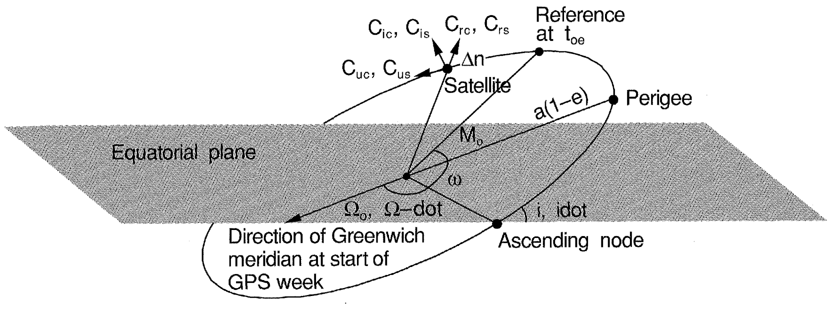

Figure 2. The parameters of the GPS broadcast ephemeris

The GPS receiver takes the ephemeris parameters and computes the coordinates of the satellite in an earth-centered, earth-fixed (ECEF) coordinate system. The details of this computation have been carefully spelled out by the designers of the Global Positioning System and can be found in a number of reference publications including the *Guide to GPS Positioning* published by Canadian GPS Associates. The particular ECEF system used by GPS is the World Geodetic System 1984 (WGS 84) of the Defense Mapping Agency (DMA). North Americans should realize that for all intents and purposes, the reference frame of WGS 84 and the new North American Datum of 1983 (NAD 83) are identical.

A new set of orbital parameters is computed for each one-hour period using overlapping data spans of four hours. A GPS satellite broadcasts the appropriate set of parameters during a particular one-hour interval.

Fresh orbital data are uploaded to the satellites up to three times daily. The initial Block II satellites can store the navigation messages for the following 14 days and therefore are afforded a certain degree of autonomy should something happen to the OCS. This autonomy capability has been extended to 180 days for the Block IIA satellites.

The broadcast ephemeris is computed with sufficient accuracy to guarantee the design goal of horizontally positioning a GPS receiver with an accuracy of 16 meters. This is the accuracy of the Precise Positioning Service (PPS), the service afforded to authorized (primarily military) users. When GPS is fully operational, the accuracy of the broadcast ephemerides will be intentionally degraded as one of the mechanisms for implementing the policy of selective availability (SA) for the Standard Positioning Service (SPS) available to civilian users. (See the "Innovation" column in the September/October 1990 issue of *GPS World.*)

Already, extensive tests of SA have been carried out. PPS users will have access to the undegraded orbits through a decryption process. Civilian users will have to be content with the lower SPS accuracy unless they use the technique of differential or relative positioning. Differential positioning is far less sensitive to orbit errors than point positioning.

For some applications even the undegraded broadcast ephemerides are not accurate enough. Very high accuracy geodetic surveys and surveys carried out to study deformations of natural and man-made structures require orbits with the highest accuracy possible. Such surveys are carried out using the differential mode of positioning with a network of receivers. Each receiver records the phase of the GPS carrier signals for subsequent analysis. Because the data do not have to be processed in real time, the data analysts have the luxury of using postcomputed "precise" ephemerides rather than the predicted orbits broadcast in the satellite's navigation message. These ephemerides are computed using data from a tracking network spanning the same time interval as that of the survey.

The DMA operates a global network of five tracking stations for the purpose of computing precise ephemerides. Data from these stations are combined with those from the five OCS stations to produce the orbits. Several civilian agencies also operate tracking networks for orbit determination. The largest of these networks is the Cooperative International GPS Network (CIGNET). This global network currently consists of about 20 stations maintained by various national agencies. The collected data are transmitted to the CIGNET Data Center in Rockville, Maryland, where they are archived and made available for orbit computation and research.

In addition to its ephemeris, each satellite transmits approximate orbit information for all of the other satellites in the GPS constellation. Known as the almanac, this information can be used by a GPS receiver to determine the location of each satellite so that it can quickly acquire the signals from satellites that are above the horizon but are not yet being tracked. The almanac consists of values for computing the Keplerian motion of the satellites and only the most significant of the correction terms is provided — the change in the right ascension of the ascending node with time. The almanac can also be used with various software planning tools to predict satellite availability at a given location and time. The almanac is updated by the OCS at least once every six days.

CONCLUSION

We have looked at the orbits of the GPS satellites, we have seen how they can be described, and we have reviewed the sources of orbital information. If you would like to learn more about the orbits of GPS satellites or satellite orbits in general, consult the *Guide to GPS Positioning* or any introductory level book on orbits such as *Fundamentals of Astrodynamics* by Bate, Mueller, and White, which is published by Dover Publications.

Although Newton established the physics of orbital motion, he probably never imagined even in his wildest dreams that one day a couple of dozen man-made objects would be whizzing around the earth that, together with a small handheld box, could tell him which apple tree he was sitting under. Such is the marvel of the Navstar Global Positioning System. ■